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Journal
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Vol. 9 SEPTEMBER, 1924 Number 3

THE EMISSION OF NITROGEN AND HYDROGEN IN THE INFRARED

A. H. Pfund

The following is a preliminary note on results obtained from a study of the infrared emission of the above gases, excited to luminosity in a vacuum tube and studied by means of a rock-salt spectrometer and a vacuum thermopile.

The vacuum tube was of the ⊔⊔ type, the over-all length being 150 cm. The bore was about 10 mm and the radiation was taken out "end on" from the lower horizontal portion which was closed with a rock-salt window. The tube was excited by alternating current from a 5 K.V.A. transformer. The spectrometer, which was of the usual Wadsworth type,[1] was equipped with concave mirrors of 35 cm focal length. Immediately behind the second slit, a single junction vacuum thermopile of the type developed by the writer[2] was mounted. Usually the vacuum was maintained sufficiently high (by means of the charcoal-evacuator[3]) to withstand a potential difference, applied to the electrodes of the charcoal-evacuator, equivalent to a 10 cm alternative spark-gap. A Leeds and Northrup high sensitivity galvanometer, operated at a scale distance of 5 meters, was connected to the thermopile.

While the immediate purpose of this work was to search for a spectral series of hydrogen beyond that discovered by Brackett,[4] it was deemed advisable to test the performance of the apparatus on some gas which had been studied previously. For this purpose, nitrogen was chosen. The results obtained by Coblentz[5] are shown in Fig. 1.

[1] Pfund. Astrophys. J., *24*, p. 21; 1906.
[2] Pfund. Phys. Zeit., *13*, p. 870; 1912.
[3] Pfund. loc. cit.
[4] Brackett. Astrophys., J., *56*, p. 159; 1923.
[5] Coblentz. Investigations of Infrared Spectra. *1*, p. 6, 317.

For the present work a continuous stream of the gas was allowed to enter the tube through the usual long capillary. By drawing out the capillary a trifle too fine and by cutting down the speed of the pump by means of a stop-cock, inserted between tube and pump, the pressure in the tube could be regulated to yield the maximum emission for the voltage applied. Purification of the (atmospheric) nitrogen was effected by means of a mixture of pyrogallol and KOH through which the gas was allowed to bubble. No trace of the ordinarily vigorous CO_2 emission bands was found in the spectrum.

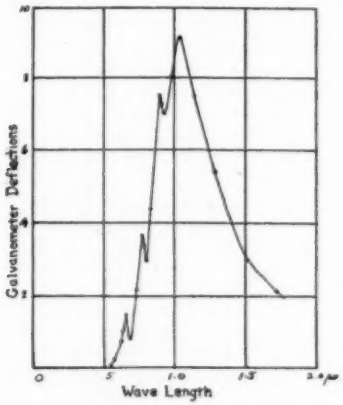

FIG. 1

The results obtained for a slit-width of 0.3 mm are shown in Fig. 2. From this it is clear that the band structure, suggested by Coblentz's results, is shown to exist strikingly. These bands are undoubtedly composed of innumerable fine lines which are not here resolved. While no immediate importance is therefore attached to the wave-lengths of the maxima or minima, it may be stated that the maxima lie at: 0.59, 0.68, 0.78, 0.90, 1.06, and 1.20μ. These values, with the exception of the last, which was not observed by him, are in substantial agreement with those of Coblentz. No prominent lines or bands were observed at greater wave-lengths.

Since these bands extend only slightly beyond 1μ, their detailed structure might be revealed by infrared photography. This work is to be taken up shortly.

For the study of hydrogen, the same apparatus was used. Both spectrometer slits, however, were widened to 1.5 mm. In consequence

of the smaller dispersion and greater thermopile sensitivity, a general sensitivity several times as great as that employed by Brackett was obtained (deflections as large as 250 mm were realized for Hα). A qualitative survey of the spectrum revealed the prominent Paschen[6] and Brackett[7] lines. After much labor, the tube was finally brought to the "black" stage[8] and the region between 5 and 10μ was explored for new lines. This region offers difficulties because of the strong radiation from the walls of the discharge tube. Furthermore, electrostatic

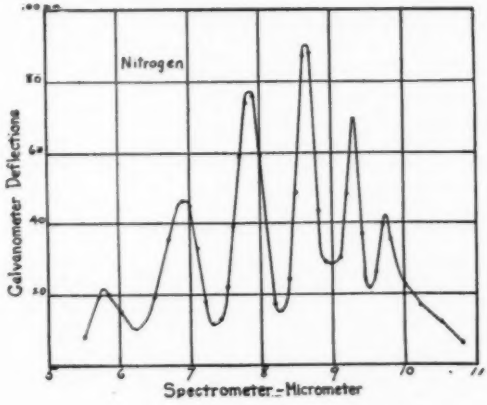

FIG. 2

disturbances, due to the starting and stopping of the discharge made the reading of small deflections difficult. The procedure finally adopted was to run the tube continuously and to record galvanometer readings at various spectrometer settings.

A typical series of deflections is plotted in Fig. 3. Superposed upon the vigorous continuous spectrum of the hot tube radiation is a small maximum, at 7.40μ. This maximum appeared only when the tube was in the "black" stage i.e. in the absence of the secondary spectrum. A careful search for a similarly situated maximum in the radiation from the hot tube (in the absence of discharge) gave negative results. It is therefore certain that the maximum in question is due to the radiating gas itself.

[6] Paschen. Ann. d. Phys., *27*, p. 537; 1908.
[7] Brackett. loc. cit.
[8] Wood. Phil. Mag., *42*, p. 729; 1921.

Upon the basis of Bohr's theory, a calculation was carried out for the wave-length emitted upon the falling back of an electron from the 6th to the 5th ring. The precise formula used was:

$$\frac{1}{\lambda} = 109{,}677.7\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad n_1 = 5\,,\ n_2 = 6$$

and the resulting wave-length of this line came out as $\lambda = 7.46\mu$. In view of the fact that the observed maximum was found at 7.40μ—no great accuracy being claimed for wave-length measurements—it seems probable that the line found is the first member of the new series represented by the above formula.

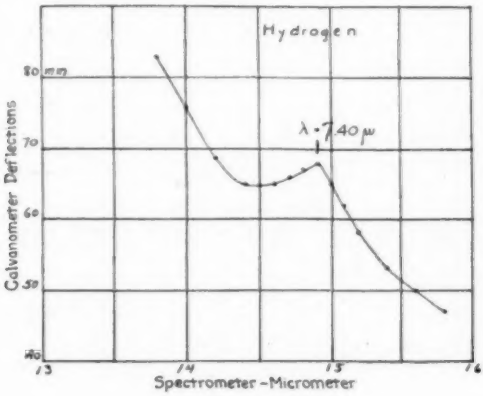

FIG. 3

For the sake of completeness, the wave-lengths of the first lines of the various series belonging to hydrogen are here recorded:

Series	X-ray designation	λ	n_1	n_2
Lyman	*K*	0.1216 μ	1	2
Balmer	*L*	0.6563 μ	2	3
Paschen	*M*	1.875 μ	3	4
Brackett	*N*	4.05 μ	4	5
New	*O*	7.46 μ	5	6

JOHNS HOPKINS UNIVERSITY,
MAY, 1924.

ON THE EFFECT OF SMALL PARTICLES IN THE VITREOUS HUMOR

By A. A. Michelson

The appearance of "floating motes,"[1] as they may be called for want of a better designation, is doubtless a familiar phenomenon though of so fleeting a character as to escape notice unless the attention is directed to its observation. This is rather difficult on account of the fact that the "motes" are very rarely in the field of the fovea, and the attempt is accompanied by the exasperating effort to catch them, notwithstanding the knowledge that the faster the eye moves in this effort the faster do the motes fly out of the field of vision.

This indicates that the appearance is due to the impression produced on the retina by the diffraction pattern produced by the presence of small particles in the vitreous humor. Their appearance is reproduced in Fig. 1. The number and sharpness of the motes visible may be con-

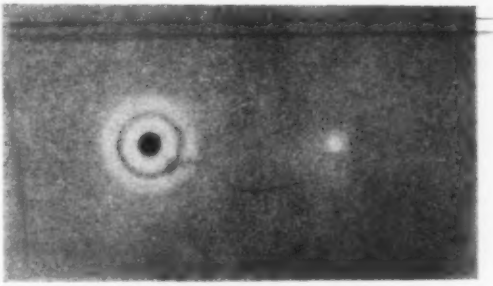

Fig. 1

siderably increased by placing a small aperture close to the pupil. The appearances observed under these conditions are roughly reproduced in Fig. 2.

Much more definite results may be obtained by observing a bright point source (sun's image in a convex mirror) through a short focus lens placed so that the image of the source coincides with the center of the pupil. The patterns observed in this case are concentric circular fringes very similar to Newton's rings, Fig. 3. The angular diameter is, however, very small (from two to ten minutes) so that it is not easy to

[1] Since writing this paper I am informed that the phenomenon described is well known under the term muscae volitantes.

identify colors; but it is clear that the centers of the ring systems are always bright and that the inner borders of the dark rings are red and the outer, blue. The diameters of the rings can only be roughly estimated and (so far as such estimates are of any value) appear to vary

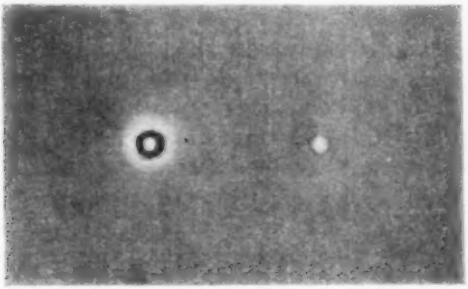

FIG. 2

as the square root of the number of the ring. The total number of rings visible depends on the "visibility" of the ring system, the fainter and larger systems showing as many as four or possibly five, while the clearer and smaller systems may show only one.

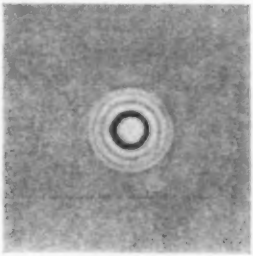

FIG. 3

On the hypothesis that these appearances are caused by minute particles suspended in the vitreous humor, illuminated by a small source producing diffraction figures on the retina, the following approximate calculation shows that most of the phenomena can be accounted for.

Let S be a point source, p the diffracting particle, and qr the retina, $pr = h$. If the particle acts by opacity alone the vibration at q will be

$$y = \int_h^\infty \frac{dS}{\lambda\rho} \cos(nt - m\rho) - \int_{\rho_1}^{\rho_2} \frac{dS}{\lambda\rho} \cos(nt - m\rho) .$$

The first integral extends over the entire wave-surface except the small area, πr^2, of the particle, and is practically $y_1 = \sin(nt - mh)$. The second extends over the surface, πr^2 of the particle, and if the distance h is large compared with the radius of the particle, this will be $y_2 = \frac{\pi r^2}{\lambda h} S(\omega) \sin(nt - m\rho)$. $S(\omega)$ is Airy's integral which, for values of ω less than 2, may be replaced by $e^{-\omega^2}$ where $\omega = \frac{\pi r}{\lambda} \alpha$.

FIG. 4

The resultant intensity of the diffraction figure will be approximately

$$I = 1 + A^2 + 2A \cos \pi h \alpha^2 / \lambda$$

where $A = \frac{\pi r^2}{\lambda h} e^{-\omega^2}$.

The visibility of the diffraction pattern,

$$V = \frac{I_1 - I_2}{I_1 + I_2} = \frac{2A_0}{1 + A_0^2}.$$

or, if A_0 is small,

$$V = 2A_0 = \frac{2\pi r^2}{\lambda h}.$$

The visibility varies directly as the area of the particle and inversely as the distance from the retina.

If the hypothesis is correct the same formulae should apply to the diffraction figures observed in a microscope when the diffracting particle

is out of focus. The results of a series of observations agree fairly well with the calculation except for values of h which are not large compared with r, as was to be expected.

Following is a table of results of a series of observations of the distances h which gave a visibility of 0.50 with particles of various sizes:

r	h	$V=\frac{2\pi r^2}{\lambda h}$ ($\lambda=0.00055$)
.0015	.05	.51
.0020	.12	.38
.0025	.10	.70
.0035	.29	.48
.0040	.50	.36

The mean is $V=0.49$, showing that the assumptions leading to the expression for V are essentially correct.

The angular semi-diameter of the bright rings as viewed from the particle is given by $\alpha^2=2n\lambda/h$, whence the visual angle will be $\theta=\frac{h}{f}\alpha$ in which f is the focal length of the eye, whence

$$\theta_n=\frac{1}{f}\sqrt{2n\lambda h} \qquad n=1,\ 2,\ 3\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot$$

The semi-diameter of the dark rings corresponding to $n=1/2, 3/2, \ldots$ will be in the proportion $\sqrt{1}$, $\sqrt{3}$, $\sqrt{5}$, etc., which agrees with observation.

The number of rings observed will be limited by the falling off of intensity with ω. If the limit of visibility corresponds to $(1/e)^2$, $\omega=1$, whence

$$\alpha=\frac{\lambda}{\pi r}=\sqrt{2n\lambda/h},$$

and

$$\bar{n}=\frac{\lambda h}{2\pi^2 r^2}=\frac{1}{\pi V}.$$

If θ_1, the angular semi-diameter of the first ring, and V, the visibility, be given, the distance and size of the particle may be found. Thus if $\theta_1=.001$ (which is about the average value observed) and $f=25$ mm, $\lambda=.00055$ mm, then from $h\lambda=f^2\theta^2$, $h=1.1$ mm, the distance of the particle from the retina. If V, the visibility, is 0.5, $r^2=f^2\theta^2V/2\pi$, giving for the radius of the particle, $r=0.007$ mm.

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CHICAGO ILLINOIS.

THE INTRINSIC INTENSITY AND POLARIZATION OF LIGHT AFTER PASSAGE THROUGH DEEP SLITS

By L. P. Sieg

Some three years ago A. T. Fant and myself[1] made a brief investigation of the intrinsic intensity possessed by light after passage through deep slits or channels between polished steel walls. A rough calculation indicated that our results agreed fairly well with theory. Later Mr. C. R. Smith[2] under my direction undertook a similar problem in which work he used, in addition to steel, Cu, Ag, Au, and Ni for the walls of the deep channels, applying these latter substances to the original steel by successive electrodeposition. He obtained distinct variations among the different metals used, which at the time of performing the experiments had not been checked by any theory. In order to obtain a check on Smith's results, and in order to investigate the phenomenon of polarization arising from the passage of light through these deep channels, I have somewhat revised and extended the simple theory given previously.[3]

The only similar work that I can discover, and which only remotely resembles the present problem, is found in two papers by the late Lord Rayleigh.[4] In these papers the passage of waves through perfectly conducting, infinitely thin slits, whose breadth, $2b$, is of the order of, or less than, the wave-length of light, was discussed. That problem, as Rayleigh states, is extremely difficult, so difficult in fact that he had to resort to rather laborious numerical calculations in order to arrive at his results. The present problem is even less amenable to exact rigorous calculation. However, a fairly rough, though apparently physically sound method, that agrees fairly well with experiment, where there have been experiments to test the matter, is here discussed. In addition the theory enables one to predict the per cent of polarization of the transmitted light. It is hoped in the near future to test this latter aspect by direct experiment.

Referring to Fig. 1, L denotes the depth of the channel, w its width, θ an angle of diffraction of a pencil of light from a random point in the

[1] Sieg, L. P. and A. T. Fant, J. O. S. A., *5*, p. 218; 1921.
[2] Smith, C. R., J. O. S. A. and R. S. I., *7*, p. 1069; 1923.
[3] Sieg and Fant. Loc. cit.
[4] Rayleigh. Roy. Soc. Lond. Proc., A, *89*, p. 194; 1913-14. Phil. Mag., *43*, p. 259; 1897

opening of the slit, and m the number of reflections of this pencil before emergence. It is fairly obvious that with a given angle of diffraction not all the light over the width of the slit will suffer the same number of reflections. However, in any given case, a certain portion of the wave will have m reflections, while the remainder will have $m+1$ reflec-

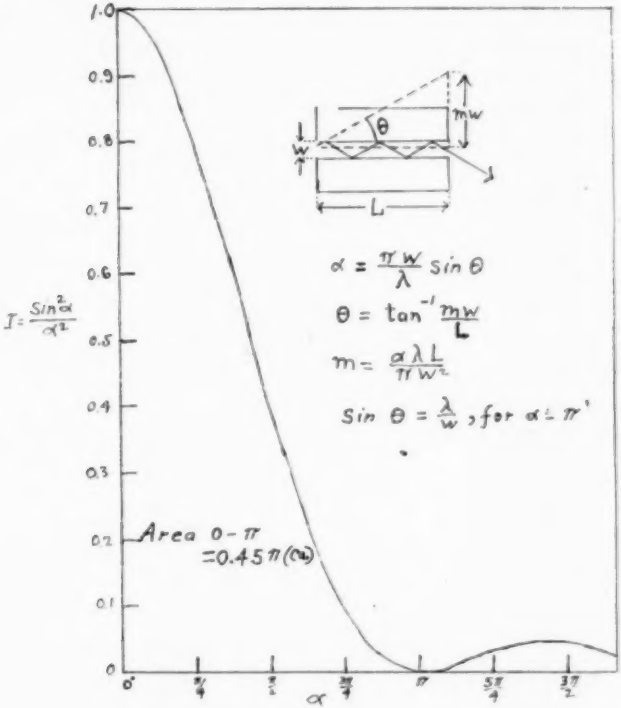


FIG. 1. *Intensity of light diffracted from a narrow slit. Diagram of deep slit, showing relations among width and depth of slit, the angle of diffraction, and the number of reflections.*

tions. The relative intensity of light diffracted in a certain direction is given by the well-known expression, $\sin^2 \mathfrak{a}/\mathfrak{a}^2$, where $\mathfrak{a}$ is an auxiliary angle given by

$$\mathfrak{a} = \frac{\pi w}{\lambda} \sin\theta \tag{1}$$

where λ is the wave-length of the light. It is easily seen that

$$\theta = \tan^{-1}\frac{mw}{L} \tag{2}$$

since θ is usually small one can substitute tan θ for sin θ and we have from equations (1) and (2),

$$m = \frac{a\lambda L}{\pi w^2} \tag{3}$$

In the same Fig. 1, we have the usual curve where the relative intensity I is plotted against the angle a. The area under the curve from 0 to ∞ is 0.5π, and from 0 to π is approximately $.45\pi$. Hence if we let a vary merely from 0 to π we shall be accounting for most of the total intensity of the diffracted light. The limiting angle θ corresponding to $a = \pi$ is easily seen from (1) to be given by

$$\sin\theta = \frac{\lambda}{w} \tag{4}$$

At this point, the solution of the problem of finding the intrinsic intensity and the state of polarization of light passing through a typical deep slit can best be explained by considering two typical examples. Suppose we are dealing with a slit between steel jaws, $w = 0.0001$ cm, $\lambda = .00006$ cm and $L = 0.1$ cm. From (3) we find, for $a = \pi$, that $m = 600$. The extreme diffracted light thus suffers 600 reflections. It would appear at first thought that no light could suffer 600 reflections and still possess any intensity, but we must remember that we are dealing with angles of incidence approaching grazing incidence, and that for such angles the reflecting power of even ordinary reflectors is nearly unity. To be sure, in this case we easily see that we cannot go as far as 600 reflections but we shall use as many as 96. The results are arranged as shown in Table 1.

TABLE 1

$\frac{a}{\pi}$	m	$\theta°$	$\phi° = 90 + \theta°$	h	$R_\perp$	R_{II}	$I_\perp = h.R_\perp$	$I_{II} = h.R_{II}$
0	0	0	90	0.5	1.00^{0}	1.00^{0}	.500	.500
.02	12	0.75	89.25	0.49	$.99^{12}$	$.883^{12}$	.441	.111
.04	24	1.50	88.50	0.48	$.983^{24}$	$.772^{24}$	.329	.001
.06	36	2.25	87.75	0.48	$.98^{36}$	$.685^{36}$	.226	....
.08	48	3.00	87.00	0.47	$.97^{48}$	$.613^{48}$	.111	....
.10	60	3.69	86.31	0.46	$.966^{60}$	$.553^{60}$	.058	....
.12	72	4.43	85.57	0.45	$.96^{72}$	....	.023	....
.14	84	5.17	84.83	0.44	$.95^{84}$	....	.006	....
.16	96	5.91	84.09	0.44	$.942^{96}$	....	.001	....

The column headed a contains certain values of a from 0 to such a value (in this case $.16\pi$) that will not yield too large a value for m. Under m we have the number of reflections corresponding to the various

values of α. ϕ is the angle of incidence corresponding to the angle of diffraction, θ. h is 1/2 the mean ordinate of the curve of Fig. 1. It is a measure of the intensity of the light diffracted through the angle θ. The half ordinate is selected, because it is assumed that half the incident intensity is due to light with the electric vector parallel to the length of the slit, and half perpendicular to the length. On viewing it in another way, half the light has its electric vector perpendicular to the plane of incidence, and half parallel to it. The columns headed R and $R_{\shortparallel}$ represent the reflecting powers of steel at the given angles of incidence for the electric vector perpendicular and parallel respectively to the plane of incidence. These values for R are obtained from the well-known expressions[5]

$$R_{\perp}=\frac{n^2(1+\kappa^2)-2n\cos\phi+\cos^2\phi}{n^2(1+\kappa^2)+2n\cos\phi+\cos^2\phi} \tag{5}$$

$$R_{\shortparallel}=\frac{n^2(1+\kappa^2)\cos^2\phi-2n\cos\phi+1}{n^2(1+\kappa^2)\cos^2\phi+2n\cos\phi+1} \tag{6}$$

These values of R for steel for three wave lengths, 0.325μ, 0.600μ, and 1.5μ are plotted in Fig. 2, for certain angles of incidence. In equations (5) and (6), n represents the index of refraction, κ the coefficient of absorption, and ϕ the angle of incidence. The columns of Table 1 headed $I_{\perp}$ and $I_{\shortparallel}$ are formed by multiplying together the appropriate values of h and R. Specifically, the number .329 in the column under $R_{\perp}$ means that for this particular pencil, diffracted at 1.5°, there were 24 reflections; the coefficient of reflection is .983, and the half ordinate of the curve of Fig. 1 is 0.48. Thus this particular pencil of light emerges with an intensity given by $0.48\times.983^{24}$, or .329. The corresponding emergent intensity of the pencil vibrating in the plane of incidence is only .001. Thus the relatively strong polarization of this particular pencil is accounted for. Now we can look upon the numbers in the columns headed I and I as ordinates of two curves, the abscissas being given by the values of α. The ratio of the sum of the areas under these two curves from $\alpha=0$ to the highest value of α used to the area under the curve of Fig. 1 will give the intensity of the transmitted light relative to that of the incident light. Again the ratio of the difference of the areas under the two I curves to their sum will give the fraction of polarization of the transmitted light. Resorting to our example in Table 1, we can get the areas under the I curves sufficiently well by

[5] Winkelmann, Handb. d. Phys. 1st Ed. p. 824.

taking the means of the ordinates in pairs, multiplying each mean by the base (in this case $.02\pi$) and summing the products; or what amounts to the same thing, summing the means, and multiplying the sum by $.02\pi$. Performing this operation we have $\Sigma[I_\perp] = 1.454$ and $\Sigma[I_\parallel] = 0.366$, where the brackets indicate that means are used. Multiplying each of these by $.02\pi$ we have the two relative intensities

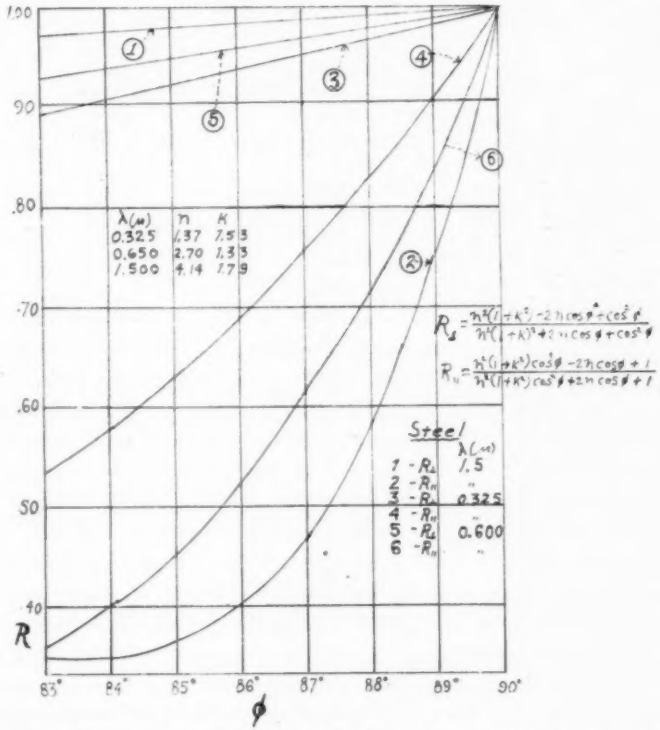


FIG. 2. *The coefficients of reflection of steel for large angles of incidence, for the electric vector both perpendicular, and parallel to the plane of incidence. List of optical constants for which these reflection coefficients are calculated.*

$.0291\pi$ and $.0072\pi$ respectively. Adding these and dividing by 0.45π, the area under the curve of Fig. 1 from $a = 0$ to $a = \pi$, we get .081 or a transmission of 8.1% of the incident intrinsic intensity. Dividing the difference of $.0291\pi$ and $.0072\pi$ by their sum, and expressing in per cent, we get a percentage of polarization of 60.3. This same method is followed for all cases where m is large. Where m is small the procedure

is somewhat different, as will be illustrated by the following case, the details of which are given in Table 2.

Again dealing with steel, let $L=0.0025$ cm, $w=0.0001$ cm, and $\lambda=0.00006$ cm. We find, as shown above, that m for $\alpha=\pi$ is 15, and θ for this value of α is 36.9°.

TABLE 2.

α/π	m	$\theta°$	$\varphi°$	h	$R_\perp$	$R_\parallel$
0	0	0	90	0.5	(1.000)°	(1.000)°
0.1	1.5	3.69	86.31	0.46	$.5(.968)^1+.5(.968)^2$	$.5(.550)^1+.5(550)^2$
0.2	3.0	7.38	83.62	0.43	$(.926)^3$	$(.340)^3$
0.3	4.5	11.07	78.93	0.36	$.5(.890)^4+.5(.890)^5$	$.5(.280)^4+.5(280)^5$
0.4	6.0	14.76	75.24	0.28	$(.863)^6$	.....
0.5	7.5	18.45	71.55	0.20	$.5(.833)^7+.5(.833)^8$	.....
0.6	9.0	22.14	67.86	0.13	$(.805)^9$	.....
0.7	10.5	25.83	64.17	0.07	$.5(.780)^{10}+.5(.780)^{11}$	.....
0.8	12.0	29.52	60.48	0.03	(.755)12	.....
0.9	13.5	33.21	56.79	0.005	$.5(.730)^{13}+.5(.730)^{14}$	.....
1.0	15.0	36.90	53.10	0	0	.....

Table 2 is not complete for it is intended to illustrate merely the variation in procedure from that discussed in connection with Table 1. For example under the column headed "$R_\perp$", the second line, we note that we have to deal with $m=1.5$, i.e., one and one-half reflections. This is equivalent to saying that 0.5 of the incident wave front suffers one reflection, and 0.5 of it two reflections. Had m been 2.37 we should have had 0.63 of the wave front reflecting two times and 0.37 of it reflecting three times. The remainder of the procedure is now clear. We obtain the value as listed under the R columns, multiply each by the proper h, and form the $I_\perp$ and $I_\parallel$ columns (not shown in the table). Then the areas are found under these two I curves as before. The result for this case gives a transmitted intensity of 43.2%, and a percentage of polarization of 51.2. The reflecting coefficients for the various angles of incidence, ϕ, are calculated by formulas (5) and (6). The reflecting coefficients for certain large values of ϕ for the metals Cu, Au, Ag, Ni, Fe, and Si are plotted in Fig. 3. On this same figure are listed the per cent transmission and per cent polarization for all these substances where in each case the wave-length is 0.6μ, $L=1$ cm, and $w=.001$ cm. It will be noted what appears as obvious, that the greater the difference between the $R_\perp$ and $R_\parallel$ coefficients, the greater the percentage of polarization, whereas the percentage of transmission is proportional to the absolute magnitudes of $R_\perp$ and $R_\parallel$. Thus a slit

having silicon jaws would produce the greatest polarization, gold the least; and silver and silicon, of the substances tested, would represent the extremes of transmission.

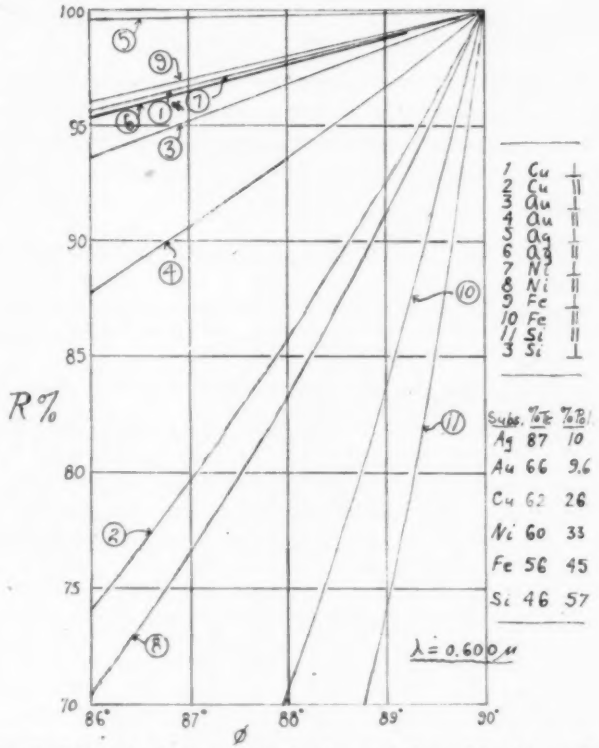
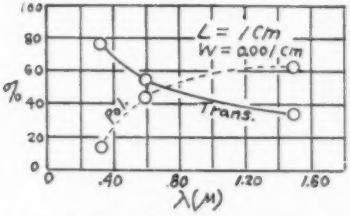


FIG. 3. *Coefficients of reflection with the electric vector both parallel and perpendicular to the plane of incidence for the substances Cu, Au, Ag, Ni, Fe, and Si. Also table of per cent transmission and polarization for these substances for a given deep slit.*

It is interesting to compare the variation of polarization and transmission, with the separate variation of the three variables, λ, L, and w. These variations for steel are shown respectively in Figs. 4, 5, and 6. In Fig. 4, we note that the polarization increases with the increase of wave-length in the range chosen, whereas the transmission decreases. We note (Fig. 5) a similar variation in these two quantities when L is the variable. If one wanted, for example, to design a deep slit for large polarization coupled with a fairly large transmission, he would have

to choose some intermediate length. Large intensity and high per cent of polarization are not simultaneously consistent. In Fig. 6 we note that the polarization decreases rapidly with increasing width, whereas the transmission increases from 0 to its final constant value of unity, only after a certain relatively large width of jaws.



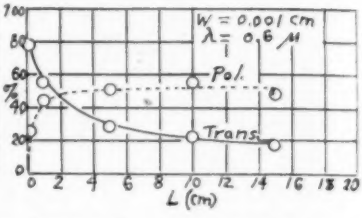


FIG. 4. *The transmission and polarization with a given deep slit, as the wave-length λ is varied.*
FIG. 5. *The transmission and polarization with a given deep slit, as the depth L of the slit is varied.*

The results for transmission as indicated in Figs. 5 and 6 have been well verified[6]; those of Fig. 4 for transmission are not in agreement with experiment. No experimental results for polarization have been obtained. It is hoped to try these soon to see if they will check the above calculations. Recently Smith[7] performed some experiments in this laboratory, the results of which should, if our theory is correct, be com-

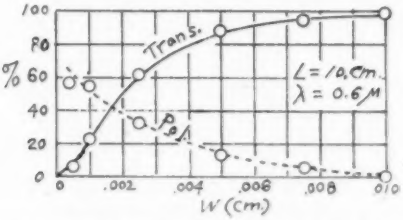


FIG. 6. *The transmission and polarization with a given deep slit, as the width W of the slit is varied.*

parable with the results for transmission listed in the table shown in Fig. 3. The agreement between these is not very satisfactory. It must be noted, however, that the calculated results are all based on certain optical constants for these various metals. Our tables of optical constants show wide variations in these constants among the results of different observers. Dealing as we do with large angles of incidence,

[6] Sieg and Fant. Loc. cit.
[7] Smith. Loc. cit.

the reflecting coefficients must be very sensitive to conditions as they are no doubt very much dependent upon the state of the reflecting surface—its polish, contamination, scratches, etc.

This problem has probably but little practical importance. It does lend us a bit of caution in measuring polarization and intensity of a source of light by means of any apparatus by which the light is made to pass at any point through a fine or a deep slit, as this slit itself introduces its own selective changes in both intensity and polarization.

THE UNIVERSITY OF IOWA,
MAY, 1924.

Measurement of the Birefringence of Optical Glasses.—In spite of the annealling to which it is subjected optical glass always retains some birefringence. It is advantageous to be able to define and measure this phenomenon.

The specimen to be measured is prepared in the form of a plate with parallel sides. The two rectilinear vibrations which traverse the plate without alteration have, on emerging, a certain path difference between them. If this difference of path is

$$\Delta = p\lambda$$

the quantity p is a simple number which can be directly determined by experiment. If n and n' are the two indices corresponding to the two vibrations and e the thickness of the plate $\Delta = (n' - n)e$ or $n' - n = p\lambda e$. It is this quantity that defines the birefringence of the substance in the direction considered, and it is deduced at once from the value of p given by experiment. If the measurement is made with white light one adopts for λ the value 0.56μ.

A study of various methods was made with the apparatus arranged as follows:

1. A source of light, such as a strong incandescent bulb diaphragmed, or a mercury arc provided with a yellow filter which then gives a light sufficiently monochromatic.
2. A polarizer of the Glazebrook type with normal faces.
3. The plate of glass to be studied carried on a support permitting of rotation about the optical axis of the apparatus and the determination of its orientation on a divided circle.
4. The apparatus for measurement.
5. An analyzer of the same type as the polarizer and provided with means for determining its orientation.

The following methods were studied:

1. *Babinet Compensator.*—One reads the position of the black fringe without the slab of glass in place, and then measures the displacement of the fringe on introducing the slab. Good results are obtained when the birefringence is strong but for the weak birefringence of good optical glass the precision is insufficient.

2. *Quarter Wave Plate.*—One places the slab of glass so that the privileged directions of vibration are at 45° to the incident vibration, and the quarter wave plate so that one of its axes shall be parallel to one of the same vibrations. The analyzer is then set for extinction, and one reads the angle through which it must be turned for extinction when the slab is removed. For weak birefringences the rotation is small and the method lacks precision.

3. *Compression Compensator.*—This is a carefully prepared block of glass in a holder with which definite pressure may be imposed on the glass. The relation between pressure and wave retardation is determined experimentally. Such a compensator is very sensitive for measurement of small values.

4. *Quartz Plate cut Perpendicular to Axis.*—This is used as a compensator by measuring the rotation from the normal plane required to compensate the birefringence of the slab. For desirable characteristics the plate of quartz must be very thin.

Preference is expressed for the compression compensator. [Henri Boissier, Revue d'Optique, *2*, pp. 107-113; 1923.] G. W. MOFFITT

Detection and Study of extremely thin Films of Silver on Glass and on Quartz.—These films are produced by heating a platinum wire, initially covered (by electrolysis) with a silver film 2 to 4μ thick, and heated by current until the silver atoms have all evaporated off. Those which pass through a hole 50 to 200μ in diameter, pierced in a screen 1.5 cm distant, are received on a glass or quartz plate 1 cm beyond the screen. The thickness of the (central spot of the) film formed on the plate is calculated from the thickness of the original film and from the geometry of the system, assuming that the silver atoms depart from the wire surface in all directions according to the Lambert cosine law, and that on striking the plate they all adhere to it without reflection. The plate is then placed in a 1-2% solution of hydroquinone diluted with some "gummi arabicum," to which a few drops of a 1% silver nitrate solution are added. After a few minutes the silver film becomes visible; the more distinctly the thicker it is. The thinnest films actually discerned by this method are calculated to be .025mμ thick; films .019 mμ thick were made but did not become visible on the plates. Development by Langmuir's device of exposing the plates to Cd vapor was also tried, and may make yet thinner films visible but was found less convenient. Estermann examined films, formed in a similar manner upon quartz, with an ultramicroscope. Films of "thickness" 0.3 $\mu\mu$, i.e. one-tenth of a monomolecular layer, showed distinct aggregations of molecules into long narrow crystals, some of which are photographed. It is deduced that when atoms are deposited from a vapor upon a solid surface, they wander around over the surface until several form into a group; for which there is other evidence.—[J. Estermann and O. Stern, Hamburg; ZS. f. Phys. Chem., *106*, pp. 399-402; 1923. J. Estermann; ibid. pp. 403-406.] KARL K. DARROW

THE VISIBILITY OF THE SPECTRUM*

By Selig Hecht**

Anyone acquainted with the field of physiological optics knows that there exists a staggering mass of literature describing different phases of the subject. The number of published papers runs easily into tens of thousands. It is therefore with much trepidation that one presumes to write a paper concerned with the phenomena of vision. My hope however, in doing so, is not to add to the existing material, but rather to subtract from it. If one can succeed in bringing a number of scattered phenomena into orderly arrangement, one decreases by so much the welter of unrelated facts that constitutes the present literature of visual science. What has given me the courage to try this is the simplicity of the ideas with which I propose to deal.

The existing theories of vision, particularly the more recent ones, are too inclusive. In attempting to explain everything, they have succeeded in explaining nothing. If it were possible to reach the eager makers of new theories of vision, I should plead with them to cease spinning webs of shimmering words—even such fashionable words as quantum and photoelectric,—and urge them to turn their energies to the painstaking investigation of visual phenomena under conditions calculated to answer definite and simple questions. It would certainly seem that more is to be gained by the rigorously quantitative treatment of a small field in a concrete manner than by the multiplication of vaguely general and obviously inadequate theories of which there are already far too many.

The present paper is an attempt to deal with certain simple phenomena of vision in terms of some elementary principles of photochemistry. No effort will be made to bewilder the reader by presenting to him the entire structure of modern physics. Neither the experimental facts to be presented nor their photochemical implications involve any but the simplest concepts.

I

One of the most familiar facts in the physiology of vision is that the different colors of the spectrum possess different degrees of brightness.

* This article contains the substance of an evening lecture delivered at Woods Hole. References to the original literature and the methods and data of the experiments will be found in a recent article in the Journal of General Physiology.

** National Research Fellow.

Nearly everyone, on looking at an ordinary spectrum which shows the usual violet, blue, green, yellow, orange, and red will say that yellow is its brightest color, and that the colors on either side become less and less bright as one looks toward the ends of the spectrum. These differences in brightness persist even when care is taken to secure a spectrum in which all the colors possess exactly the same content of radiant energy. This has been known for many years, and indeed has been the subject of much investigation, especially in recent years with the advent of new needs and methods for the photometry of differently colored lights. From this work it appears that the brightest part of the spectrum is the light whose wave length is about 555mμ, a point which lies in the yellow-green.

There are a number of facts, which, though closely connected with those just described, are however, hardly known outside the circle of people directly connected with the study of vision. Perhaps the most striking of these is the existence of a colorless spectrum. When the visible spectrum is reduced to a very low intensity, it loses its colors. This is easily demonstrated. All that is necessary is a dark room and a spectroscope arranged in such a way that the source of light is properly screened out. The spectrum is allowed to fall on a sheet of white paper, and its intensity is changed by bringing the paper nearer or farther from the ocular. As the intensity is gradually diminished, first one and then another color drops out, and finally, the spectrum, which has just been a brilliant band of beautiful hues, becomes a drab band of colorless gray.

This colorless spectrum,—colorless and spectrum sound like a contradiction in terms,—is similar to its colorful progenitor, in that it too is not uniformly bright. Its central portion is distinctly brighter than the two ends. However, even a rough examination shows that the brightest part of this colorless band is not in the same place as in the bright, colorful spectrum. Although this position of maximal brightness has been known accurately for many years in the case of the colored spectrum, it has only recently been determined accurately in the colorless spectrum. It is found to be at 511mμ a point that corresponds to blue-green in the colored spectrum. The transition between the two kinds of spectra, colored and colorless, is gradual. As the intensity is decreased, and the colors become dim, the maximum of brightness shifts gradually from the yellow-green toward the blue-green, and when the colors finally disappear, it remains in the now colorless

blue-green. This gradual shift was observed many years ago by Purkinje, and has been named after him the Purkinje effect.

There are two questions that may be raised with regard to these curious facts. The first asks why it is that the spectral colors are not uniformly bright. Why, for example, does a yellow patch of light always seem brighter than a blue or a red, even if care is taken to see that both patches radiate the same amount of energy into the eye? Moreover, why, even when no colors are visible, do the different portions show up in different degrees of brightness? In short, what is there in the visual mechanism that accounts for the brightness of the spectrum? The second question is almost a corollary to the first. What is responsible for the difference in the position of maximum brightness between the bright, colored spectrum and the dim, colorless spectrum? In other words, what is the cause of the Purkinje effect?

Before attempting to explain the curious phenomena just described, it is necessary to acquaint the reader with certain of their quantitative aspects. It would be interesting to be able to say how much brighter one color is than another. Unfortunately the eye is incapable of doing this. The eye can judge that one color is brighter than another but it cannot tell how much brighter. The eye, however, can tell quite well when two colored patches are of equal brightness. The expedient has therefore been adopted of measuring the relative energy emitted into the eye by two patches of color which the eye judges to be of the same brightness. Similarly, with the colorless spectrum, measurements are made of the relative energy radiated into the eye by two portions of the spectrum which appear to be of identical brightness.

In this way, results like those in Fig. 1 are secured. The left hand curve gives the results for the dim, colorless spectrum, while the right hand curve gives them for the bright, colored spectrum. For convenience the amount of energy necessary to secure a given brightness at the point of maximum brightness is made equal to 10 units in both curves. The "colorless" curve shows how much energy relatively is necessary for any portion of the spectrum to produce a given brightness sensation in the eye. The same is true for the "colored" curve. The actual brightness is of course not the same in the two cases; it is very much greater in the colored spectrum,—being way beyond the brightness that gives a maximum Purkinje effect. The curves simply present graphically and quantitatively the facts which were described qualitatively above, and prepare the way for the considerations which follow. For reasons that will become apparent, it will be expedient to consider first the visibility of the colorless spectrum.

II

There is a dictum as old as photochemistry itself that light can be effective only when it is absorbed. Our retinas must therefore possess a substance which absorbs the light of the visible spectrum. It is the light energy absorbed by this substance which produces the chemical processes necessary to set off the impulse in the optic nerve. Let us assume that in order to produce a given sensation of brightness, it is necessary that this substance should absorb a given amount of energy. This is an almost self evident assumption, because it merely says that

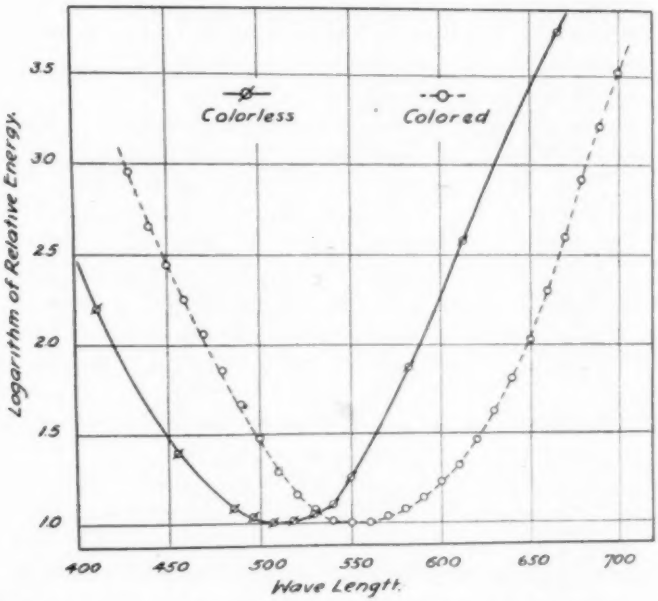


FIG. 1. *The relative energy necessary in different parts of the spectrum to produce a given brightness, at very low intensities (solid curve) where the spectrum is colorless, and at high intensities (broken curve) where the spectrum possesses its usual colors.*

to produce the same amount of photochemical activity, the same amount of outside light energy must be absorbed by the sensitive substance of the retina. Let it be recalled that we are dealing with a colorless spectrum, in the vision of which there is only one variable in sensation,—brightness. Colors are not visible at all, only differences in brightness.

There is another dictum in photochemistry which states that a substance does not absorb equally well the light from the different parts of the spectrum. Some substances absorb only the red; others only the green; others still different portions. As a result we have the variagated colors and shades of natural objects and dyes. There are a number of ways of describing the powers of absorption of a substance. The simplest is in terms of an absorption coefficient a which is merely the ratio of the absorbed light I_a to the incident light I_i. This gives that

$$a = \frac{I_a}{I_i}$$

where $a = 1$ when absorption is complete, and a is a fraction less than 1, when absorption is only partial.

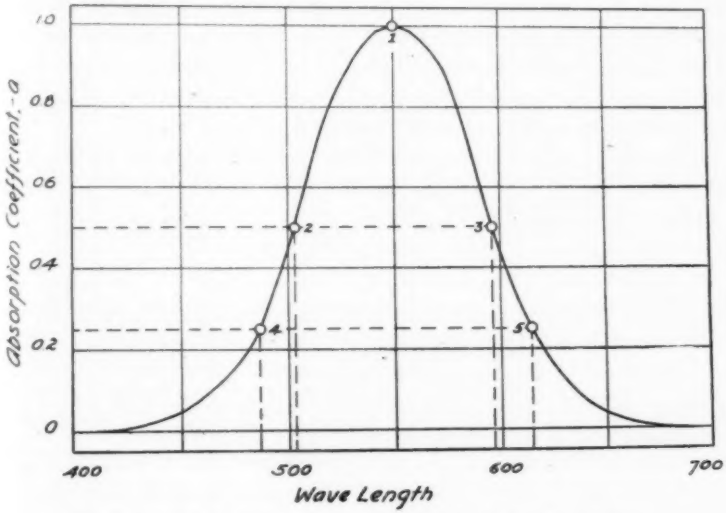


FIG. 2. *General shape of a simple absorption band in the visible spectrum.*

It is possible to measure the absorption of a substance for light from every portion of the spectrum. When this is done for substances possessing a simp'e type of absorption, one gets data which may be represented by Fig. 2. On the horizontal abscissas we have the spectrum, and on the vertical ordinates we have the degree of absorption, or absorption coefficient corresponding to every part of the spectrum. The absorption in this imaginary case is maximal at the point 1 and tapers off at either side to almost nothing. Such absorption curves

are quite simple and are very common in all classes of substances and in all parts of the spectrum, even in the ultraviolet and in the infrared. The substance in the retina which absorbs the light from the visible spectrum may probably have an absorption curve similar to that in Fig. 2.

We have assumed that in order to cause a definite brightness sensation, this substance must absorb a given amount of energy from the outside light. Call this amount of energy E. On referring to Fig. 2, it is apparent that at the point 1 in the spectrum, there will be absorbed 100 per cent of the incident light. Therefore, in order for the substance to absorb E units of light, it will be necessary for the incident light to be only E units also. At points 2 and 3, however, the case is different. Here $a=0.5$, and only 50 per cent of the incident light is absorbed. Therefore if E units are to be absorbed by the substance in order to produce the given brightness, it will be necessary for the incident light to have an intensity of $2E$. Similarly, at the points 4 and 5, $a=0.25$, and the absorption is only 25 per cent; and if E units are to be absorbed at this wave-length, the incident light will have to be $4E$.

Referring to the formula given above for the absorption, we may now write that $E=I_a$, that is, that the absorbed light is equal to E. The equation then becomes,

$$a=\frac{E}{I_i}$$

which in words means that the absorption coefficient of the substance in the eye is inversely proportional to the incident light necessary for the absorption of a given amount of energy E. The absorption of the constant amount E will produce a constant brightness sensation. Therefore, since E is constant, the absorption coefficient of the sensitive substance in the eye will be proportional to the reciprocal of the amount of energy needed to produce a given brightness in the eye.

We possess in Fig. 1, the experimentally measured amounts of light which are necessary to produce a given brightness sensation. Therefore, if our reasoning has been correct, the reciprocal of these amounts of incident light should give us a curve similar to that of Fig. 2, representing the absorption spectrum of the sensitive substance in the retina. The left hand curve in Fig. 3 shows that this actually happens. The curve possesses the familiar shape of the absorption spectrum of numerous well known and carefully investigated substances. The individual curves for these substances vary not only in extent—some being nar-

rower than others,—but particularly in position on the spectrum. Their general shape is, however, the same.

We are now in a position to answer our first question. The reason that portions of the spectrum differ in brightness is due to the fact that the sensitive substance in the eye has such an absorption curve that some parts of the spectrum are more readily absorbed than others. Referring to Fig. 3, it is plain that both ends of the visible spectrum are much less absorbed than the center. Therefore, even though the spectrum may be arranged to be equal in energy throughout, more of

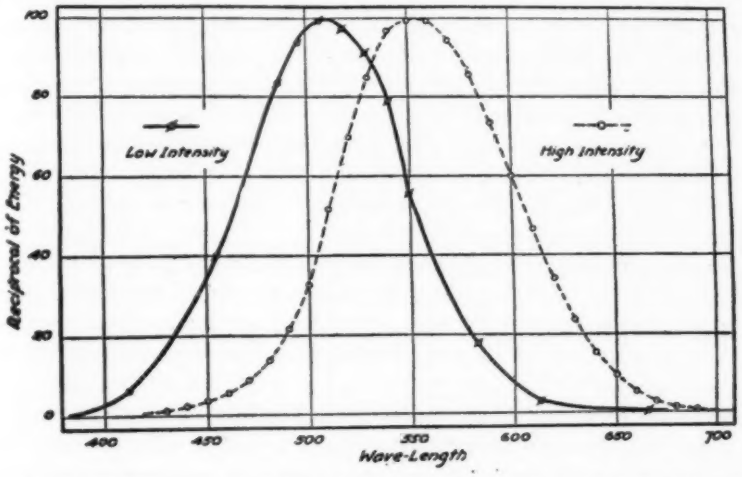


FIG. 3. *Visibility curves for colorless and color vision. The ordinates, which are the reciprocals of those in Fig. 1, are proportional to the absorption coefficients of the sensitive substances in the retina. The low intensity curve is therefore the absorption spectrum of the sensitive substance in the rods, while the high intensity curve is the absorption spectrum of the sensitive substance in the cones.*

that energy will be absorbed in the center at 511mμ than anywhere else; and since more energy absorbed means more chemical action in the retina, the center of the spectrum will be the brightest portion, the other parts varying in brightness depending on their respective absorption.

III

If we consider the brightness of the colored spectrum from the same point of view as we have just considered the colorless spectrum, we arrive at a surprising fact. Taking the reciprocals of the data in Fig. 1 for the colored spectrum, we get the curve to the right in Fig. 3. Even

on superficial examination the two curves of Fig. 3 appear to be similar. Careful testing shows them to be practically identical. They differ merely in position, the "colored" curve being an average of 48mμ farther toward the red than the "colorless" curve.

This identity of the two curves is quite significant. The ideas by means of which we arrived at the curve of the colorless spectrum involve the notion of the presence in the retina of a *single* substance possessing a simple absorption band. In the brightness of the colored spectrum we find a curve which is identical with the colorless spectrum curve. The obvious conclusion would seem to be that the brightness perception of the colored spectrum, too, is an expression of the photochemical properties of a single substance possessing a simple absorption band in the visible. Let it be emphasized that nothing has been said about the perception of color. That is still in the deepest mystery, though the recent work of Weigert has offered some stimulating suggestions. So far we speak only of the sensation of brightness produced by the colored spectrum. And it seems reasonable to conclude that this brightness, irrespective of color, depends on the absorption properties of a single substance in the retina.

The measurements with the colored spectrum are made with the centrally located fovea of the retina, which contains only cones. It has been known for years that these cones contain the machinery for the seeing of colors. Therefore it is in the cones that we will find the sensitive substance, which is responsible for the brightness of colors, and whose absorption band is shown in Fig. 3. Such a substance must be in the cones. In order to see, certain photochemical processes must take place in the cones; therefore, energy has to be absorbed; and there must be a substance which will absorb this energy in such a way as to give the curve in Fig. 3.

IV

Having answered the first question, namely, why it is that different portions of the spectrum normally produce different sensations of brightness, let us consider the second question. This is concerned with finding the reason for the difference in position of the maximum brightness in the colored and colorless spectrum. The measurements of the brightness of the colorless spectrum are made with the rods of the eye. Therefore the substance concerned with the absorption of light of the colorless spectrum is in the rods. We thus have two mechanisms concerned with brightness perception. One is in the cones and one is

in the rods. The absorption maximum of the one is at 555mμ, and the absorption maximum of the other is at 511mμ, in the conditions in which they occur in the cones and rods respectively. It is curious that these two mechanisms should show practically identical absorption curves, differing merely in position by about 48mμ.

There are three logical possibilities for the relationship of the sensitive substances in these two mechanisms. They may be widely different substances; they may be very similar substances differing slightly in the position of some group in the molecule; they may be the same substance. The first possibility of complete divergence does not seem sensible. Substances which have similar functions in organisms are generally similar chemically, for example, the hemoglobins and chlorophylls from vastly different groups. The second possibility of near relationship is very plausible. If it were true,—and our data give us no means of deciding for or against it,—it would be merely a significant fact to record and to spectulate upon. The third possibility of identity however, offers some rather interesting and to a certain extent, stimulating situations.

According to this idea we have the same substance showing an absorption band which is in two different positions depending on whether the substance is present in the rods or in the cones. Are such substances known, and what are the conditions which control the position of the maximum of absorption?

Some fifty years ago Kundt noticed that the refraction of a solvent has a distinct influence on the position of the absorption band of a substance dissolved in it. Since his time this work has been extended and the relation between the refractive index of a solvent and the position of the band of a dissolved substance has been confirmed. In its most general form Kundt's rule may be stated as follows. If one colorless solvent has a greater refracting or dispersing capacity than a second, then the absorption bands of a substance dissolved in the first will be nearer the red end of the spectrum than when dissolved in the second. Kundt's rule has had a particular field of support in the sensitizing effects of certain dyes on the photographic plate. It was found that the maximum of sensitivity of the photographic plate does not correspond with the maximum of absorption of the added dye. The maximum of sensitivity is distinctly shifted toward the red. Careful work by Eder showed that this shift is due to the actual position of the absorption maximum of the dye in the condition in which it is dissolved in the silver bromide grains. Eder obtained shifts toward the red varying

from 15mμ to as high as 65mμ. The refractive index of a silver bromide grain is very much higher than that of water,—which therefore makes the phenomena correspond to Kundt's rule.

What has already been said is enough to show how the same substance could give the two bands in Fig. 3, 48mμ apart, depending on the refractive index of the medium in which it is dissolved. The rods and cones are two distinct media. It is therefore conceivable that the cones are more refractive than the rods, and that the same substance dissolved in the cones would have its absorption band shifted strongly toward the red in comparison with its position in the rods.

V

There is an interesting experimental possibility which follows from the association of Kundt's rule with the visual mechanism. In the rods there is a photosensitive substance which is present in sufficient concentration to be visible in the isolated retina, and even in the living eye under proper conditions of observation. This substance is known as visual purple. As present in the retina and when extracted from it, visual purple shows a remarkable sensitivity to light. Undoubtedly visual purple is concerned with the visual functions of the rods. If this is true, the absorption spectrum of visual purple should be identical with the one which we derived from the brightness characteristics of the colorless spectrum for the hypothetical sensitive substance in the rods. This is almost a logical necessity,—but not quite. The terminal segments of the rods, in which the visual purple is dissolved, are fairly dense, and highly refractive bodies,—much more so, for example, than water. If we may argue from Kundt's rule, the position of the absorption maximum of visual purple in the rods will not be the same as that in solution in water. The absorption band of visual purple in the rods should be shifted toward the red, because the medium is much more refractive than water. The brightness sensitivity of the rods to the colorless spectrum should, therefore, just like the sensitivity of a stained photographic plate, be farther toward the red than would be expected from the position of the absorption spectrum of visual purple as measured in water solution.

The experimental evidence supports this expectation to the fullest extent. The absorption spectrum of visual purple in solution has been measured several times. The most accurate of the measurements are those of Köttgen and Abelsdorff. They are presented in Fig. 4. The continuous curve with the small circles is the brightness visibility curve

of the colorless spectrum taken directly from Fig. 3, and according to our reasoning represents the absorption spectrum of visual purple in the condition in which it is dissolved in the rods. The other points and the broken line are the data of Köttgen and Abelsdorff for the visual purple of the monkey and the rabbit. It is apparent that the absorption spectrum of visual purple in the rods is definitely and consistently farther toward the red than in water solution. The two curves are, however, identical, and represent no doubt the same substance in two different media obeying Kundt's rule.

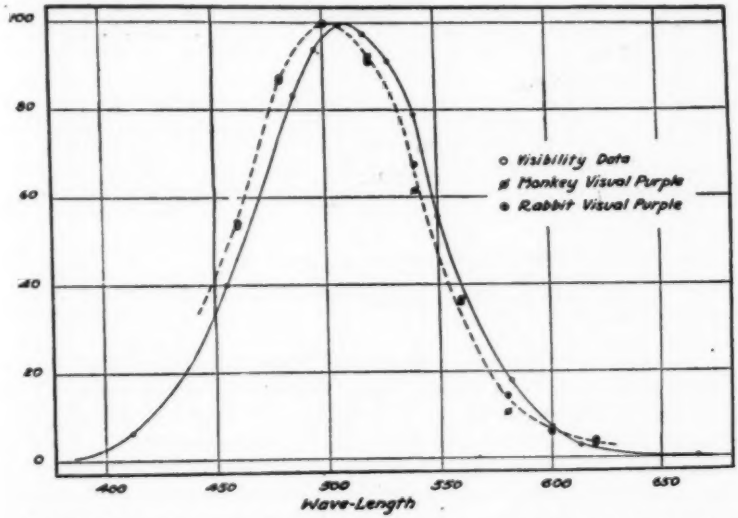


FIG. 4. *Relation between the absorption spectrum of visual purple in watery solution and the absorption spectrum of the sensitive substance in the rods as given by the visibility curve in Fig. 3. Though the two curves are obviously identical, the visibility curve is shifted 7 or 8 mμ toward the red, as would be expected in terms of Kundt's rule.*

I have studied carefully all the published experiments relating to the visibility of the colorless spectrum and the absorption spectrum of visual purple in solution. In every case the results show this shift of about 7 or 8mμ, and always in the same direction, toward the red. In going over the literature I found one apparent exception to this in Fig. 1 of Parsons' book on Color Vision. On examining the source of the curves in Trendelenburg's original article, it became apparent that Parsons had accidentally mislabelled the two curves. When properly labelled, these data show the shift in the same direction as all others. Its truth is therefore not to be questioned.

VI

The answer to the two questions which were raised in the beginning of this paper can now be expressed simply in terms of an hypothesis. The brightness of the spectrum,—its visibility, in short,—depends on the relative absorption of spectral light by a substance in the eye. This photosensitive substance is visual purple. Its absorption spectrum in watery solution has a maximum at 503mμ. Dissolved in the more refractive rods, its maximum becomes shifted to 511mμ, and in this situation it controls the brightness of the colorless spectrum. Dissolved in the hypothetically still more refractive cones, the absorption maximum shifts to 555mμ, and in this condition controls the brightness of the colored spectrum.

I have suggested this as an hypothesis to correlate the relevant facts in a simple and concrete manner. The usefulness of this hypothesis however will depend mainly on the results of experiments which follow directly from it. Some of these experiments are now being carried out.

DEPARTMENT OF PHYSICAL CHEMISTRY,
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Comparison of Electron Emission from Incidence and Emergence Sides of a thin metal Film irradiated by X-rays.—Thin films of platinum deposited by cathode sputtering on one face of a thin plate of celluloid or carbon (0.4 mm thick) were suspended in a highly evacuated tube, connected to an electrometer, irradiated by 78000 volt x-rays incident alternately on the two faces of the plates, and the rate of charging up of the electrometer determined in each case. The electrons were gathered up by a cylindrical collecting electrode almost entirely enclosing the plate, and charged alternately to the same potential as the plate and to 50 or 100 volts below it. The thickest film was estimated as being 15mμ thick, by an (inaccurate) optical process; the thinnest film took 1/20 as long in being sputtered and was assumed to be 1/20 as thick. More electrons are emitted on the emergence side than on the incidence side, the maximum value of the ratio being 1.14; the disproportion is accentuated when only electrons faster than 100 volts are allowed to escape from the film, being as high as 1.33. The ratio approaches unity as thinner films are substituted, which may mean that it is essentially a difference in initial speed and not in initial quantity of electrons emitted in the two directions from the atoms. A few measurements on Al and Cu give the same result.—[W. Seitz; Ann. d. Phys. *73*, pp. 182-189; 1924.]

K. K. DARROW

ABERRATION OF LIGHT IN TERMS OF THE THEORY OF RELATIVITY AS ILLUSTRATED ON A CONE AND A PYRAMID*

By Vladimir Karapetoff

ABSTRACT

A ray of light is propagated in the XY plane and its angle with the X-axis is determined by two observers, S and S', who are in relative motion to each other along the X-axis at a velocity q (expressed as a fraction of the velocity of light). If the angles measured by them are δ and δ', then it is proved that $\tan\frac{1}{2}\delta/\tan\frac{1}{2}\delta' = \cos(45°+\frac{1}{2}\alpha)/\cos(45°-\frac{1}{2}\alpha)$, where $\sin\alpha = q$. This relationship is interpreted geometrically by means of a cone in oblique coordinates. The axes of coordinates are X, Y, and time T. The T and X axes are different for the two observers, and the four axes T, X, T', X', form a Lorentzian Plane to which Y is perpendicular. The properties of the cone are considered in detail and a mechanical model of the cone, built of wood and iron, is described. At a desired velocity q, one can compare directly the angles δ and δ'. It is also shown that the relationship between the angles δ, δ', and α can be represented on a pyramid. References are made to other articles by the same writer, in which oblique axes are also used for the solution of problems in the theory of relativity.

Sometime ago the present writer showed that in the simplest case of a uniform motion of a point along the direction of a uniform relative motion of two observers, the Lorentz-Einstein equations of transformation of coordinates correspond to a change from one set of oblique coordinates to another. He has also built a two-dimensional mechanical model on which this fundamental relativity transformation is directly conveyed to the eye. In this model time and space are used as oblique coordinates.[1] Following a similar line of reasoning, the author has also shown that the relativity transformation of the vectors of electric and magnetic intensities in a plane electromagnetic wave can be conveniently represented in oblique coordinates.[2]

1. Statement of the Problem

In the present article it will be shown that it is possible to represent a ray of light (or a point moving at the velocity of light) by means of a three-dimensional construction in Euclidean space, in oblique coordinates, using time as the third dimension. This gives a cone as a locus, and knowing the angle at which one of the observers sees the ray, the corresponding angle for the other observer can be immediately drawn or computed. In other words, the problem of aberration of light in

* Presented before the American Physical Society, Washington, D. C., April 25, 1924.

[1] For a popular description of this model see *Science and Invention*, *11*, p. 442; 1923.

[2] *The Physical Review*, *23*, p. 239; 1924.

terms of theory of relativity has been reduced to a simple geometric relationship. A three-dimensional model has been constructed on which this relationship is directly conveyed to the eye.

The problem of aberration is shown schematically in Fig. 1. Two observers, S and S', move at a uniform relative velocity q with respect to each other, along the XX' axis. They observe a ray of light in the XY plane, common to both. The S observer finds the ray, R, to be at an angle δ to the line of motion XX', while the S' observer finds *the same ray* to be R', at a different angle, δ', to XX'. The problem is to find the relationship between δ and δ', for a given q.

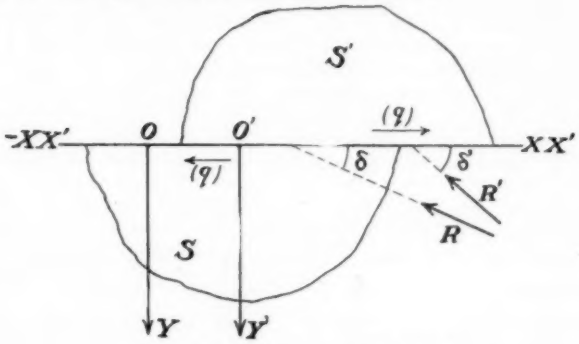

FIG. 1. *The problem of aberration of light.*

2. THE LORENTZIAN PLANE

Let x, y, t be the space and time coordinates for the S observer, and let x', y', t' be those for the S' observer. Then the usual Lorentz-Einstein equations of transformation are

$$t = \beta[t' + (q/c^2)x'] \tag{1}$$

$$x = \beta(x' + qt') \tag{2}$$

$$y = y' \tag{3}$$

$$\beta = c/(c^2 - q^2)^{0.5} \tag{4}$$

where c is the velocity of light.[3] We shall take $c = 1$ and shall introduce an angle α such that

$$\sin \alpha = q \tag{5}$$

where the velocity q is now expressed as a fraction of the velocity of light. Eqs. (1) and (2) then become

$$t \cos \alpha = t' + x' \sin \alpha \tag{6}$$

$$x \cos \alpha = x' + t' \sin \alpha \tag{7}$$

[3] See, for example, O. W. Richardson, *The Electron Theory of Matter*, p. 300; 1916.

These equations have the following simple geometric meaning (Fig. 2). Let OX and OT be the space and time axes used by the S observer to plot his results, the angle between these axes being $90°+\alpha$. Let X' and T' be the corresponding axes for the S' observer, at an angle of $90°-\alpha$. Let OB be the common or *universal bisector* of both sets of axes. When the relative velocity q equals zero, the two sets of axes coincide and become an orthogonal system X_0OT_0, at 45° to OB. As q increases, the axes OX and OT move outward, while OX' and OT' approach OB.

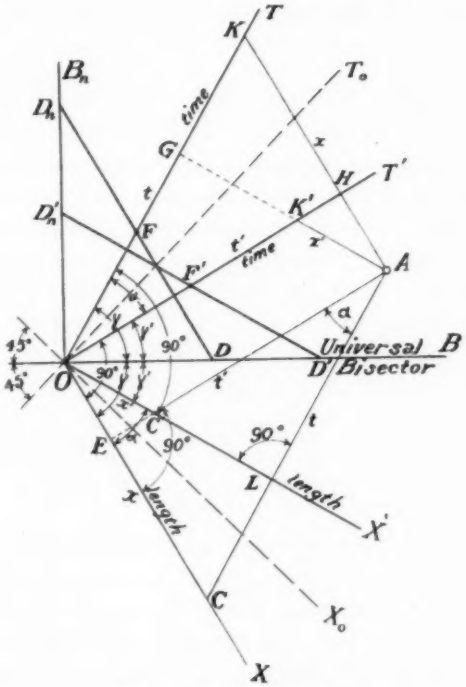

FIG. 2. *The Lorentzian Plane.*

We shall furthermore assume that the scales for time and for distances are the same, so that if one centimeter represents a unit of length, it also represents a unit of time.

Consider a point A whose coordinates are x, t, x', t'. We have

$$EA = C'A + EC' \tag{8}$$

$$OL = OC' + C'L \tag{9}$$

It will be readily seen from the right triangles that these relations are identical with eqs. (6) and (7), so that the two systems of coordinates in Fig. 2 are in accord with the fundamental equations of restricted relativity. This means that an "event" which is correctly represented by point A for the observer S, is also correctly represented by the same point for the S' observer.

The plane containing the foregoing four axes has been named by the author the *Lorentzian Plane*. It is understood of course that this plane is only a "computing" plane in which the results of observation are plotted, and not the plane in which the phenomenon of aberration actually takes place.

3. The Universal Bisector OB is a World Line of a Point Moving at the Velocity of Light

It is of interest to note that OB in Fig. 2 is the space-time locus of a point moving at a velocity of light along the $X\,X'$ axis (Fig. 1). Lay off OF =unit time and draw FD parallel to OX. From the geometry of the figure, FD =unit distance, the triangle OFD being isosceles, so that $OF = FD = 1$. Therefore, for any point on OB, $x = t$, and the velocity $x/t = 1$. Since we have assumed the velocity of light $c = 1$, OB represents the space-time locus of a point which for the S observer is moving at the velocity of light and which passes through the origin O at the instant $t = 0$. Drawing now the triangle $OF'D'$ for the S' observer we satisfy ourselves that for him also OB is the time-space locus (the world line) of a point which to him seems to move at the velocity of light.

We thus arrive at the familiar paradoxical statement of the theory of relativity that a point which for an observer moves at the velocity of light, moves at that velocity for any other observer, no matter what the relative velocity of the second observer with respect to the first. Instead of accepting this statement as a fundamental assumption of the theory of relativity, we have deduced it from the foregoing arrangement of the axes of coordinates in the Lorentzian Plane. Of course, this is no proof of the proposition itself, since the axes have been originally selected to include this result.

For points moving at the velocity of light in the negative direction, the *negative bisector*, OB_n, is the space-time line. The length FD_n on the velocity line is equal to FD.

4. THE BISECTOR CONE

The Lorentzian Plane interprets only eqs. (1) and (2), and in order to consider also eq. (3) we shall add a Y-axis perpendicular to the Lorentzian Plane. According to eq. (3), values of y are the same as of y', so that this axis is common for both observers. All the axes for both observers are shown in Fig. 3, the main notation being the same as in Fig. 2. Fig. 3 is a quantitative representation of the space-time relations to scale, and not merely a "qualitative picture". This figure has been drawn in an isometric projection, with OY at 135° to OB and with the dimensions parallel to OY contracted to one half of their true values. For example, the radius Ff of the semi-circle is shown equal to one-half of its true value, which is FD.

In Fig. 2 the universal bisector OB represents the space-time locus of a point moving at the velocity of light in the positive direction of the X-axis, while the negative bisector, OB_n, represents the space-time locus of a point moving at the same velocity in the negative direction of the X-axis. In both cases the point passes through the origin at the instant $t = 0$. In Fig. 3 it is necessary to represent a point moving at the velocity of light in any direction in the XY plane (Fig. 1), and passing through the origin at the instant $t = 0$. With the space representation in Fig. 3, any straight line passing through the origin is the world line of a point which moves at a uniform velocity in the XY plane (Fig. 1), with the condition that $x = y = 0$ when $t = 0$. This is because for such a line both x and y increase in proportion to t. It remains therefore to select only such straight lines for which

$$\sqrt{x^2 + y^2} = t . \tag{10}$$

This condition is a special case of the equation

$$r = ct . \tag{11}$$

where r is the distance covered by the point in the XY plane, and c is the velocity of light. In this treatment c is assumed to be equal to 1, and replacing r by its projections, eq. (10) is obtained. This equation squared gives

$$x^2 + y^2 = t^2 \tag{12}$$

which is the equation of a cone in the semi-oblique system of coordinates XYT.[4] When t is constant, that is, in planes parallel to the XY plane (Fig. 3) the cross-sections of the cone are circles with centers on the T-axis. When $t = 1$, $x^2 + y^2 = r^2 = 1$, so that the radius of the circle is then

[4] See, for example, Snyder and Sisam *Analytic Geometry of Space*, index under "Cone."

equal to the velocity of light. The semi-circle $D_n f q' D$ represents that half of the velocity circle which lies in the direction of the positive Y-axis, and has the velocity scale DD_n (Fig. 2) for its diameter. Any generator of the cone, such as ogg', represents a point moving at the velocity of light in the XY plane (Fig. 1). The angle $DFg = \delta$ is the angle between the direction of the motion of this point and the X-axis.

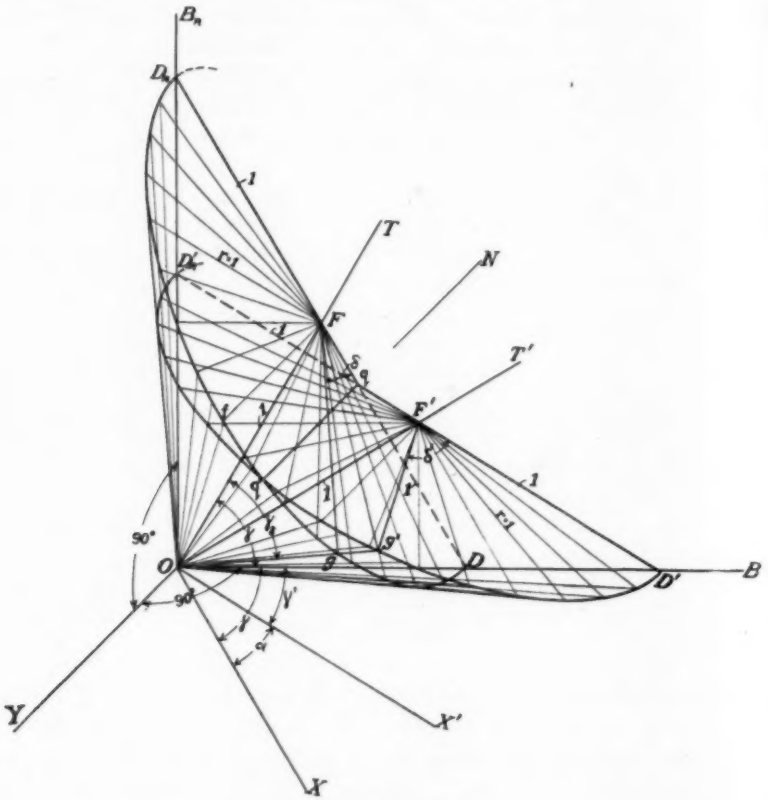

FIG. 3. *The bisector cone.*

In accordance with the fundamental principle of relativity, the S' observer should also find the velocity corresponding to the line Og' equal to 1. Hence, the cross-section of the same cone by a plane parallel to the $X'OY$ plane, at the distance $OF' = 1$, must also be a circle of radius 1 (see proof below). The semi-circle $D'_n q' g' D'$ is part

of this circle. For one and the same ray, Ogg', the S observer measures the angle $DFg = \delta$, while the S' observer finds the angle $D'F'g' = \delta'$. Thus, the cone with its two circular cross-sections gives a quantitative visual representation of the problem of aberration. This cone, as actually represented by a mechanical model, is shown in Fig. 6.

5. THE CONE REFERRED TO THE AXES B, B_n, AND Y

The equation of the cone in the XYT system of coordinates is given by eq. (12). To find the equation of the same cone in the $X'YT'$ system, we substitute the values of x and t from eqs. (6) and (7) in eq. (12). Remembering that $y = y'$, we find after reduction that

$$x'^2 + y'^2 = t'^2 \tag{13}$$

This equation is identical in its form with eq. (12) and we therefore conclude that the cross-section of the cone parallel to the $X'Y$ plane, at the distance $t' = 1$, is also a circle or radius 1.[5]

It would be a mistake, however, to consider the surface itself to be a straight circular cone, in the usual sense of the word. In order better to see the shape of the cone, we shall now transform its equation to the orthogonal system of "common" coordinates OB, OB_n, and OY. Let the ordinates along OB be denoted by b, and those along B_n by n. Then, using the familiar expressions of analytic geometry for transformation of oblique coordinates in the BOB_n plane (*not* a relativity transformation) we get:[6]

$$x \sin 2\gamma = b \sin \gamma - n \cos \gamma \tag{14}$$

$$t \sin 2\gamma = b \sin \gamma + n \cos \gamma \tag{15}$$

In these expressions the angle γ is that shown in Figs. 2 and 3, namely

$$\gamma = 45° + \tfrac{1}{2}a \tag{16}$$

The corresponding angle for the S' observer is

$$\gamma' = 45° - \tfrac{1}{2}a \tag{17}$$

Substituting expressions (14) and (15) in eq. (12), we get after reduction:

$$y^2 \sin 2\gamma = 2bn \tag{18}$$

Instead of γ, the angle a can also be used from eq. (16). We then get

$$y^2 \cos a = 2bn \tag{19}$$

This is the equation of the cone in common or bisector coordinates b and n.

[5] The foregoing proof is, of course, but a particular case of the invariancy of the expression $x^2+y^2+z^2-c^2t^2$, when $z=0$.

[6] See, for example, Seaver's *Mathematical Handbook*, p. 143, formulae 1342.

Eq. (19) being symmetrical with respect to b and n, the cone has an axis of symmetry ON, at 45 degrees to both OB and OB_n. In order to find the shape of the cross-section of the cone in a plane normal to ON, we turn the axes OB and OB_n clockwise by 45°, so that OB_n coincides with ON. If the new coordinates are b' and n', we have

$$b = (n' + b') / \sqrt{2} \tag{20}$$

$$n = (n' - b') / \sqrt{2} \tag{21}$$

Substituting these values in eq. (19), we get

$$y^2 \cos \alpha + b'^2 = n'^2 \tag{22}$$

In other words, the normal cross-section is an ellipse, and the ratio of its axes is as 1 to $\sqrt{\cos \alpha}$, the major axis being in the Y-direction. When $\alpha = 0$, the normal cross-section becomes a circle.

While the length of the major axis of the ellipse is a function of the angle α, that is, of the relative velocity q of the two observers, the two circular cross-sections are preserved, and only the angle between them changes with q. This property of the cone is taken advantage of in the model (Fig. 6) in that each semi-circle forms a frame to which the bars which represent rays of light are attached.

6. The Relationship Between the Angles γ_δ and δ

For each ray of light in Fig. 3, such as Og, there is a vertex angle $\gamma_\delta = gOT$; our next problem is to find a relationship between γ_δ and δ. These angles are shown in Fig. 4, the cone itself being omitted. The various axes and the ray Og are the same as in Fig. 3. $TCEO$ is the plane through the ray and through the axis OT. The plane gFD contains the circle $Dq'D_n$ (Fig. 3) and is parallel to the XY plane. The irregular tetrahedron $OgFD$ has three sides equal to 1. Hence, if a sphere of radius 1 be described with F as a center, the points O, g, D will lie on the sphere. Imagine therefore a *spherical triangle* with the vertices at O, g, and D. Since the plane gFD is perpendicular to $OTAX$, the angle at D in the spherical triangle is that between two mutually perpendicular diametral planes. The triangle FgO being isosceles, the angle at F is equal to $180° - 2\gamma_\delta$. The triangle FOD is also isosceles and the angle at F is equal to $180° - 2\gamma$. Applying a fundamental formula of spherical trigonometry, we have

$$\cos (180° - 2\gamma_\delta) = \cos (180° - 2\gamma) \cos \delta \tag{23}$$

or

$$\cos 2\gamma_\delta = \cos 2\gamma \cos \delta \tag{24}$$

Since the angle FgO is equal to γ_δ, the angle EOg is also equal to γ_δ, and Og bisects the angle EOT. Thus, *the cone bisects all the projection planes, such as TCEO, for all values of* δ *from 0 to 360°.*

For this reason the cone can be properly called the *bisector cone.* We have previously introduced the angle γ in the Lorentzian plane, related

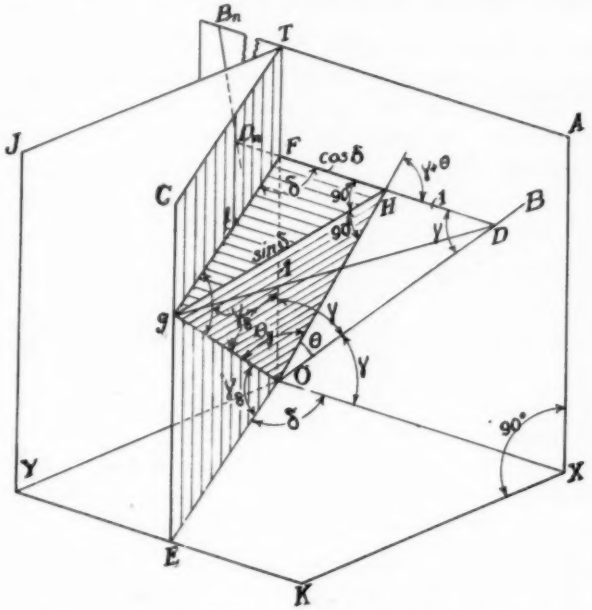

FIG. 4. *The angles which determine the direction of a ray of light.*

to the velocity angle α by eq. (16). It is now convenient to introduce a corresponding angle α_δ in the plane $TCEO$, related to γ_δ by the equation

$$\gamma_\delta = 45° + \tfrac{1}{2}\alpha_\delta \tag{25}$$

Eq. (24) may then be rewritten in the form

$$\sin \alpha_\delta = \sin \alpha \cos \delta \tag{26}$$

We thus reach the following answer to the problem stated at the beginning of this section: The cone bisects all the projection planes passing through OT; the vertex angle, γ_δ, corresponding to the direction angle δ of the ray, is expressed by eq. (24), or in a different form by eq. (26). In the TX plane the vertex angle is $45°+\frac{1}{2}\alpha$ and gradually decreases to the value of 45° in the plane $TJYO$ ($\delta = 90°$). It then

further decreases and reaches its minimum value, equal to $45° - \frac{1}{2}\alpha$, on the negative side, $B_n T$, of the Lorentzian plane.

Similar conclusions may be stated for the observer S', by replacing everywhere $+\alpha$ by $-\alpha$, and γ by γ'.

7. The Intrinsic Orientation Angles, θ and θ_y, of a Ray

In Fig. 4, the ray Og may be defined by the following two angles: angle θ which the projecting plane, OgH, makes with the bisector OB, and the angle of inclination, θ_y, between the ray and its trace OH. It is possible to obtain a simple relationship between δ and δ' by expressing both these angles through θ. From the triangle OFH we have

$$1 / \sin(\gamma + \theta) = \cos\delta / \sin(\gamma - \theta) = OH / \sin 2\gamma \qquad (27)$$

so that

$$\cos\delta \sin(\gamma - \theta) / \sin(\gamma + \theta) \qquad (28)$$

From the right triangle Ogh

$$OH = \sin\delta / \tan\theta_y \qquad (29)$$

Moreover, from eq. (16), $\sin 2\gamma = \cos\alpha$. Hence, eqs. (27) and (29) give

$$\sin\delta = \cos\alpha \tan\theta_y / \sin(\gamma + \theta) \qquad (30)$$

Eq. (28) may also be written in the form

$$\cos\delta = (\sin\gamma \cos\theta - \cos\gamma \sin\theta) / (\sin\gamma \cos\theta + \cos\gamma \sin\theta)$$

from which

$$\tan\theta = \tan\gamma (1 - \cos\delta) / (1 + \cos\delta) = \tan\gamma \tan^2 \tfrac{1}{2}\delta \qquad (31)$$

Because of the general principle of relativity, a similar equation holds true for the S' observer, so that

$$\tan\theta = \tan\gamma' (1 - \cos\delta') / (1 + \cos\delta') = \tan\gamma' \tan^2 \tfrac{1}{2}\delta' \qquad (32)$$

8. The Law of Aberration

From eqs. (16) and (17), $\sin\gamma = \cos\gamma'$ and $\sin\gamma' = \cos\gamma$, so that eqs. (31) and (32) give

$$\tan \tfrac{1}{2}\delta / \tan \tfrac{1}{2}\delta' = \cos\gamma / \cos\gamma' \qquad (33)$$

This is an expression for the *law of aberration*, or an analytical relationship between the angles δ and δ' in Figs. 1 and 4. Both γ and γ' are functions of the angle α, in other words, of the relative velocity of the two observers. When $\alpha = 0$, $\delta = \delta'$ and there is no aberration.

Usually the law of aberration is given in a somewhat different form which can be obtained by equating the middle terms of eqs. (31) and (32). Namely,

$$\tan\gamma (1 - \cos\delta) / (1 + \cos\delta) = \tan\gamma' (1 - \cos\delta') / (1 + \cos\delta') \qquad (34)$$

But from trigonometry

$$\tan \gamma = \tan (45^\circ + \tfrac{1}{2}\alpha) = \sqrt{(1 + \sin \alpha) / (1 - \sin \alpha)} \qquad (35)$$

Hence, remembering that $\tan \gamma = \cot \gamma'$, eq. (34) becomes

$$(1+\sin \alpha)(1-\cos \delta)/(1+\cos \delta) = (1-\sin \alpha)(1-\cos \delta')/(1+\cos \delta') \qquad (36)$$

or, after reduction,

$$\cos \delta' = (\cos \delta - \sin \alpha) / (1 - \sin \alpha \cos \delta) \qquad (37)$$

According to eq. (5), $\sin \alpha$ is equal to the relative velocity q of the two observers, so that eq. (37) becomes

$$\cos \delta' = (\cos \delta - q) / (1 - q \cos \delta) \qquad (38)$$

This is the form in which the law of aberration, or the relationship between δ and δ', is usually given.[7]

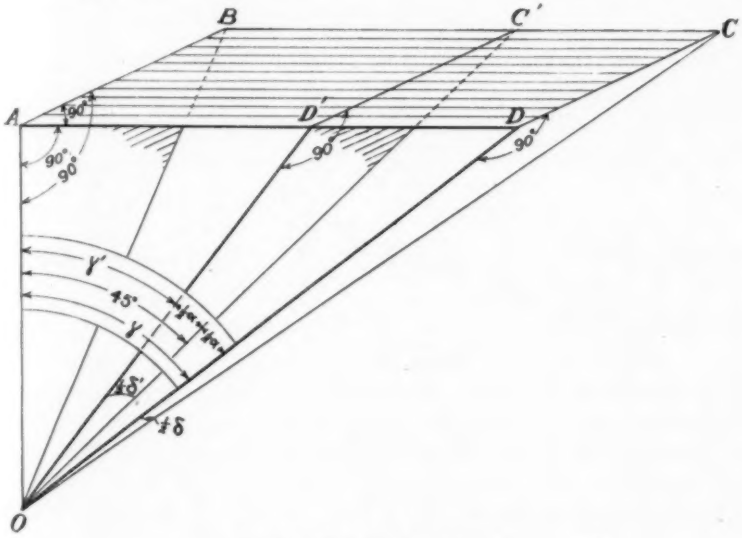

FIG. 5. *The aberration pyramid.*

9. THE ABERRATION PYRAMID

Another graphical interpretation of the law of aberration, as expressed by eq. (33), is shown in Fig. 5. *ABCDO* is a pyramid with a rectangular base and with the corner *A* like that of a cube, i.e., with three right angles. The side *OAD* is perpendicular to the base *ABCD*.

[7] See, for example, Richardson, *loc. cit.*, p. 306, eq. (14).

Furthermore, the angle $AOD = \gamma$ and $DOC = \frac{1}{2}\delta$, where γ and δ have the same meaning as before. Then, if an intermediate cross-section, $D'OC'$, be drawn, such that the angle $AOD' = \gamma'$, the angle $D'OC'$ will be found equal to $\frac{1}{2}\delta'$ and to satisfy eq. (33). For a proof we write

$$\tan \tfrac{1}{2}\delta = DC/OD ; \tag{39}$$

$$\tan \tfrac{1}{2}\delta' = D'C'/OD' ; \tag{40}$$

$$OD = OA/\cos \gamma ; \tag{41}$$

$$OD' = OA/\cos \gamma' ; \tag{42}$$

$$DC = D'C' \tag{43}$$

Dividing the first of these equations by the second, and substituting the values from the remaining three equations, eq. (33) is obtained.

10. An Approximate Formula for Aberration

When the relative velocity q of the two observers is very small as compared to that of light, angle α is also very small, and eq. (33) can be simplified as follows: In this case the angle δ' differs but little from δ and we can put $\delta' = \delta + \Delta\delta$. Eq. (33) may then be rewritten in the form

$$\frac{\tan \frac{1}{2}(\delta + \Delta\delta)}{\tan \frac{1}{2}\delta} = \frac{\tan (45° + \frac{1}{2}\alpha)}{\tan 45°} \tag{44}$$

Subtracting unity from both sides of this equation, and dividing the result by $\frac{1}{2}\Delta\delta\frac{1}{2}\alpha$, we get

$$\frac{\tan \frac{1}{2}(\delta+\Delta\delta) - \tan \frac{1}{2}\delta}{\frac{1}{2}\Delta\delta} \cdot \frac{1}{\frac{1}{2}\alpha \tan \frac{1}{2}\delta} = \frac{\tan(45°+\frac{1}{2}\alpha) - \tan 45°}{\frac{1}{2}\alpha} \cdot \frac{1}{\frac{1}{2}\Delta\delta \tan 45°} \tag{45}$$

With $\Delta\delta$ sufficiently small, the first ratio on the left-hand side of this expression approaches the value of the derivative of $\tan \frac{1}{2}\delta$ with respect to $\frac{1}{2}\delta$, that is, the value $\cos^{-2} \frac{1}{2}\delta$. For the same reason, the first ratio on the right-hand side of the equation approaches the value $\cos^{-2} 45°$. Therefore, *approximately*,

$$\tfrac{1}{2}\alpha \tan \tfrac{1}{2}\delta \cos^2 \tfrac{1}{2}\delta = \tfrac{1}{2}\Delta\delta \tan 45° \cos^2 45° ,$$

or, after simplification, and putting $\alpha = q$,

$$\Delta\delta = q \sin \delta \tag{46}$$

11. Mechanical Model

The model shown in Fig. 6 corresponds to the geometric construction in Fig. 3. The two semi-circles are shaped out of steel bars, 9.5 mm. by 2.5 mm., bent to a radius of 37 cm. The extremities of the semi-circles slide in slots mutually at right angles and representing OB and

OB_n in Fig. 3. The two wooden frames, painted red and blue respectively, represent the YX and YT planes for the two observers. The semicircles are pivoted at their mid-points to the top bars of the wooden frames. The length of the steel rod which represents the Y-axis is 45 cm. Instead of indicating values of angle a, the curved scale is

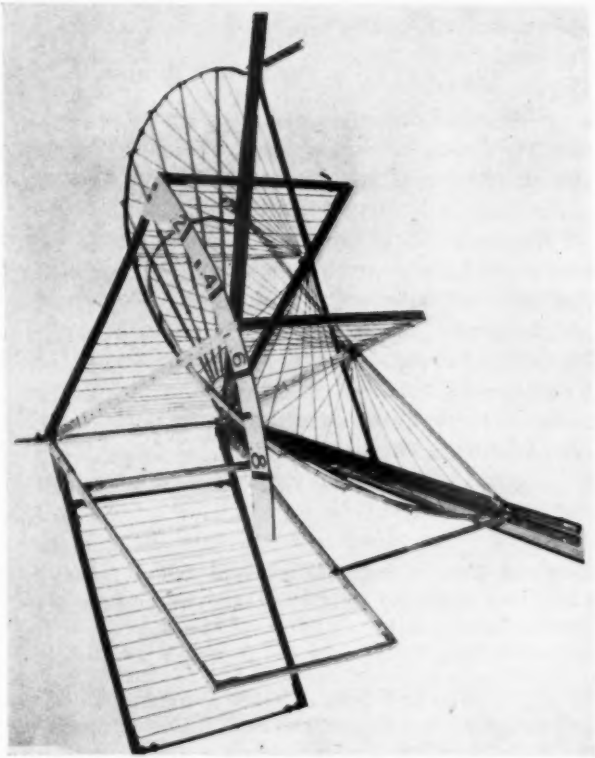

FIG. 6. *A mechanical model of aberration of light.*

divided directly in decimal fractions of the velocity of light. The device may be set by means of thumb-screws at any value of q between 0.20 and 0.80 of the velocity of light.

The steel bars which represent rays of light have a cross-section of 6.0 mm by 2.5 mm and are made each of two parts which can slide relatively to each other, to enable a setting of the device for various

values of a. The coordinate planes and the two circular cross-sections are laced with blue and red twine, so as to bring them out more clearly. All the parts referring to the S observer are painted red, and those referring to the S' observer are painted blue.

The model was constructed by two of our seniors in electrical engineering, Messrs. J. S. Ensor, Jr., and R. C. Van Sickle, to whom the author wishes to express his sincere gratitude for the valuable assistance rendered.

12. NEXT PROBLEMS TO BE CONSIDERED

The above-described cone gives such a convenient picture of straight-line motion in a plane, that it is of interest to apply the same mode of representation to some other relativity problems. Among such, the following three naturally suggest themselves:

(a) The Doppler effect; it probably can be represented by means of the same cone and a surface concentric with it.

(b) A straight-line motion of a point in the XY plane, at a velocity below that of light.

(c) The apparent change in length of a line moving in the XY plane at a velocity below that of light.

The author hopes that other investigators will extend his treatment to these and to other problems of relativity.

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Do absorbed X-rays ionize dissolved Salts?—Experiments on aqueous solutions of copper sulphate, [5g ($CuSO_4$+5ag) in 25g water] and sodium acetate [25 g (CH_3COONa+3 ag) in 25 g water], placed in thin wooden cells and irradiated by x-rays from a Coolidge-tube excited by about 60 000 volts peak voltage. The extra current (if any) due to the x-rays, which could have been detected had it amounted to as much as 10^{13} electrons/sec., was imperceptible. The amount of x-ray energy absorbed by the solutions (calculated in a rather devious manner) amounts roughly to 10^{17} times the energy required to dissociate a single molecule; so that if it had all gone into dissociating molecules into charged ions and they had all reached the electrodes the current would have been at least 10^4 times as great as it actually was. (The electrodes were however at the extreme ends of a cell 8.8 cm long and only 0.6 cm wide, and apparently the P.D. between them was less than 4 volts).—[A. Janitsky (Frankfort) ZS. f. Physik, *20*, pp. 280-287; 1923.]

K. K. DARROW

ATOMIC FORCE FIELDS*

By F. S. Brackett

Introduction

The numerous regularities exhibited in the development of the quantum defect[1] as one passes from one atom to another of higher atomic number but similar structure, indicate the existence of definite relations between the stable energy states of the atoms of different elements. Since the quantum defect is thought to arise from the penetration of the virtual orbits within the regions of greater force existing within the atom body, the physical significance of these regularities may be sought in the study of the force fields within the atom body.

In dealing with Bohr's new atomic model of interpenetrating real and virtual orbits, since it is impossible to obtain a direct solution of such a many body problem, valuable information may be gained by considering the average charge in the atom body. Confining our attention to the circular orbits, we may safely assume a radial force and treat it as constant for a given orbit. If the energy of an electron in such an orbit is known, together with the quantization of the orbit, both the size of the orbit and the magnitude of the force can be found.

The change in the energy which an atom undergoes when an electron is carried to infinity from a given orbit is known from spectroscopic data. Unfortunately other parts of the atomic system may undergo change in energy at the same time, so that this energy change can not be said to be the energy of the electron in that orbit.

By neglecting, for a moment, this consideration and applying the energy values to the electron itself, by simple theory we find for each circular orbit an "apparent force" and an "apparent radius." This apparent force will of course differ for different orbits depending chiefly upon the average number of electrons lying within the orbit. It can most easily be treated as due to an apparent charge $Z_a e$ on the nucleus, where Z_a will roughly be the difference between the atomic number and the average number of electrons within the orbit. From simple theory we have the energy

$$W = - Rh \frac{Z_a^2}{n^2}$$

* Presented at the Cincinnati meeting of the American Physical Society, December, 1923.

[1] Brackett and Birge, J. O. S. A., *8*; Feb. 1924.

Thus if an electron fall from infinity into the n^{th} orbit, energy is emitted

$$h\nu = -W$$

or

$$\nu = R\frac{Z_a^2}{n^2}$$

The frequency ν is the greatest which can be attained by a transition where the orbit of quantum number n is the final orbit. It therefore corresponds to the series head and is spoken of as the term value of the orbit. It can be found from spectroscopic or x-ray data wherever there are known series. Then

$$Z_a = n\sqrt{\frac{\nu}{R}}$$

the "denominator"

$$D = \sqrt{\frac{R}{\nu}}$$

so

$$Z_a = \frac{n}{D}$$

The value of Z_a for the virtual orbits may thus be obtained directly from the curves of the previous article. Recent values for the $\sqrt{\nu/R}$ in the x-ray region are given in tabular form in the article by Bohr and Coster.[2] Values corrected for relativity are given by Wentzel.[3] The latter values have been used for the atoms of higher atomic number where the corrections are appreciable.

The apparent radius of the orbit has the value

$$a_a = \frac{n^2h^2}{4\pi^2me^2Z_a} = a_0\frac{n^2}{Z_a} = nDa_0$$

where a_0 is the radius of the 1_1 orbit of hydrogen.

If the apparent charge Z_a is plotted against the nuclear distance, in general four points are known for each atom, from the 1_1, 2_2, 3_3, and 4_4 circular orbits respectively. Furthermore near the nucleus and thus completely within the electrons of the atom body, $Z_a = Z$, the atomic number, while completely outside the atom body all except the excited electron will have a shielding effect so that $Z_a = 1$. From these data curves may thus be drawn for any given atom representing the apparent charge as a function of the distance. It will of course be noted that the

[2] ZS. f. Physik, *12*, p. 342; 1923.
[3] ZS. f. Physik, *16*, p. 46; 1923.

radius of an orbit of any definite quantum number is fixed by the apparent charge so that to any other conceivable value of Z_a there is a corresponding radius. The locus of all points corresponding to all possible sizes of orbit for a given quantum number may thus be drawn. As these are given by the equation

$$Z_a a_a = n^2 \text{ (where the scale } a_0 = 1 \text{ is used)}$$

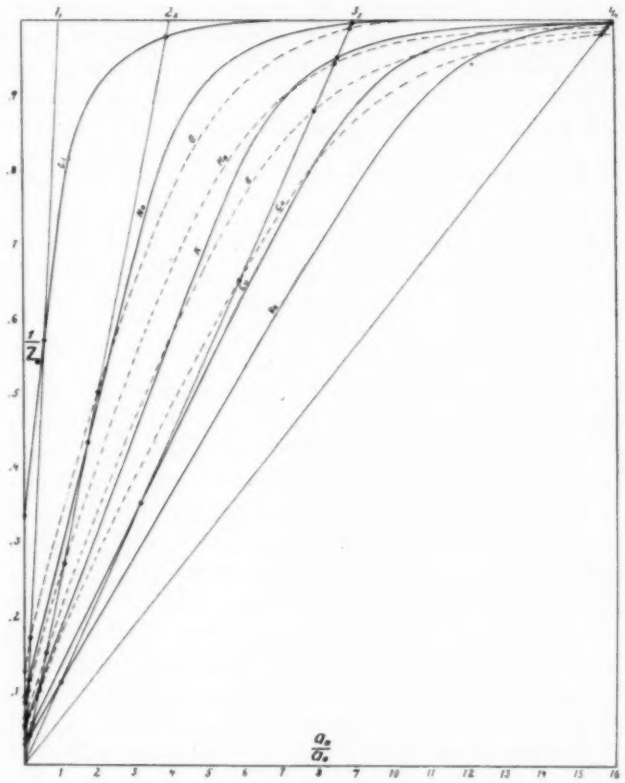

FIG. 1. *General character of field.*

the curves of possible values are seen to be equilateral hyperbolas. The curves representing the conditions in a particular atom intersect the curves of possible values for given quantum numbers in the points corresponding to existing circular orbits. From such a graph it is seen that the former curves also resemble shifted equilateral hyperbolas.

In order to test this possibility both sets of curves have been plotted, using the reciprocal of Z_a in place of Z_a.

DISCUSSION OF GRAPHS

The values of $\frac{1}{Z_a}$ have been plotted as ordinates and those of a_a ($a_0=1$) as abscissae. The curves of "possible values" for any definite quantum number now appear as true straight lines, the quantum numbers, corresponding to each straight line, being indicated at the top. The points corresponding to existing orbits have been indicated by small circles. The points for each one of the alkali metals have been connected by full line curves, those of O, Mg, S, and Ca by dotted curves. All these curves become asymptotic to $\frac{1}{Z_a}=1$ for great distances from the nucleus. The exact form of the curves can not be determined because of the wide separation of the points. It will however be seen that in the case of the alkali metals where several points are known within the atom body, they lie on a straight line which intersects the vertical axis in the points $\frac{1}{Z_a}=\frac{1}{Z}$. This is better shown in Fig. 2, where the region within the atom body has been plotted on a larger scale. The elements O, Mg, S, and Ca show a linearity in a much more limited region. It will however be remembered that the alkali metals have only completed orbital groups within the atom body, while the latter elements have in addition a wide sweeping electron which may be expected to modify the conditions for some distance within the atom.

According to our quantum assignment, two orbits have in no case been given the same quantum designation. If there were two such orbits our curves must cross the diagonals twice. Granting such to be the case there must be a third such orbit unless we abandon the demand that the force be continuous. In the case of copper the presence of an unexplained x level[4] with the characteristics of a sub 3 circular orbit, lends some support to Bohr's assignment of 3_3 to the orbit designated by us as 4_3, thus making with the x-ray orbit three such levels. The absence of such x levels in other cases however supports our assignment.

It will be noticed that the slopes of the linear segments present a regular progression from element to element. Fig. 3 shows the *reciprocal* of these slopes plotted against the atomic number. With

[4] See Fowler's "Report," p. 111.

the exception of the Li and O points where the data are insufficient to warrant any weight, the points lie on well defined straight lines, showing discontinuities of slope at the points where according to Bohr

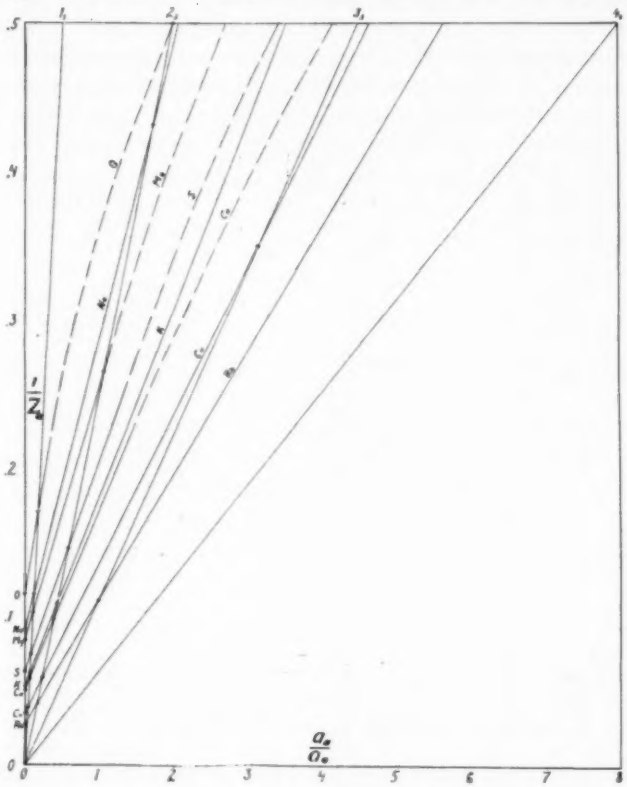

FIG. 2. *Linear relations within the atom body.*

the formation of a circular orbital group takes place. The slopes have the values:

$$c = .35$$
$$= .23$$
$$= .11\tfrac{1}{2}$$

or practically in the ratio 3:2:1.

Since the slope of a segment of a curve in Fig. 1, may be considered to represent a fundamental condition for the charge distribution in a particular atom, the constancy in certain ranges of the rate of change of

the reciprocal of the slope expresses a fundamental condition of atomic development.

In the previous paper by Brackett and Birge (loc. cit. p. 222, Fig. 1.) the graph in Fig. 4 was given, where it is seen that the deeply penetrating orbits (continuous curves) show a quantum defect (i.e. distance below horizontal lines) which varies directly with the number of orbital groups in the atom, and so with the atomic number for ranges of uniform

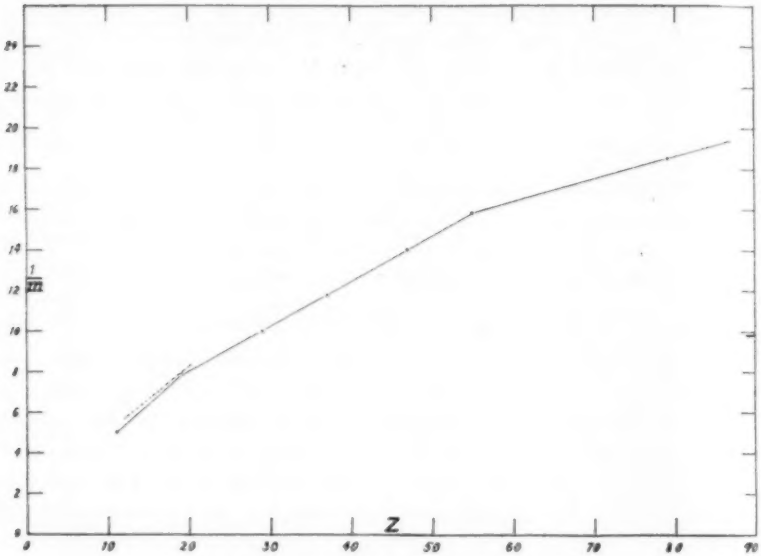

FIG. 3. *Dependence of slope on atomic number.*

development. The slope undergoes a pronounced change, however, where the inbuilding takes place. This linear variation and the change in slope at this point, it was mentioned, were general characteristics shown by all groups of elements alike. Not only does the change in slope on Fig. 4 occur at the same point as that on the previous diagram of this paper (Fig. 3), but the change is also in the ratio of 3:2 as well. A direct correspondence is therefore found between the rate of penetration of the deeply penetrating virtual orbits and a fundamental condition of the atom body. It should be borne in mind that these facts appear from absolutely unrelated data, the former from the visual spectra, the latter from the x-ray.

If we consider the atomic development in a general way, two typical stages may be defined—*first*, where the added electrons fill new orbital groups, in general temporarily complete with four electrons;—*second*, where an inbuilding takes place, enhancing groups already temporarily complete and forming a new deep lying circular group. When a first stage has run its course we might think of a saturated condition having been reached, after which a second stage must begin. The discontinuities shown in Fig. 3 occur where these saturated conditions are reached.

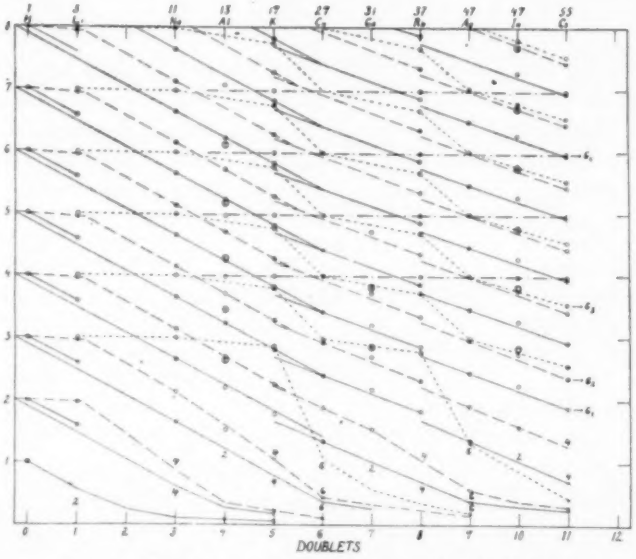

FIG. 4. *Dependence of quantum defect on number of orbital groups.*

If instead of plotting the reciprocals of the slopes of the original curves against atomic number, we plot the slopes themselves against the reciprocal of the atomic number, we find a linear relation for the "*saturated*" conditions, as shown in Fig. 5. This enables one to predict that since atomic number 87 is a saturated condition, an inbuilding should follow element 92 by analogy to the other second stages. This would however lead to a new stage of atomic development where the slope of that segment of the curve in Fig. 3 would be zero, i.e. the slope of the curves in Fig. 1 would no longer change from atom to atom. Whether this is in any way related to the fact that there are no elements of higher atomic number is as yet purely a matter of conjecture.

ANALYTICAL DISCUSSION

The linear portions in the first curves may be represented by the equation:

$$\frac{1}{Z_a} = m\frac{a_a}{a_0} + \frac{1}{Z} \qquad 1)$$

where the slope m is peculiar to the element but as has been pointed out m is related to the atomic number. From Fig. 3 we have

$$\frac{1}{m} = cZ + k \qquad 2)$$

where both c and k change with the formation of each new circular group.

The difference between the apparent number of charges on the nucleus and the atomic number may be termed the total screening effect, which Bohr has called γ. Equation 1) may be written

$$Z - Z_a = mZZ_a\frac{a_a}{a_0} = \gamma \qquad 3)$$

Further if one is simply concerned with the values at the circular orbits the relation

$$n^2 = Z_a\frac{a_a}{a_0} \qquad 4)$$

can be applied, whence

$$\frac{\gamma}{n^2} = mZ \qquad 5)$$

Thus the linearities in the curves for the various elements of the first diagram are equivalent to a constancy of $\frac{\gamma}{n^2}$ within the atom body. This condition is best realized in the case of atoms where the atom body is made up of completed orbital groups and is only approached in other cases. Then by equation 2) we have

$$\frac{\gamma}{n^2} = \frac{Z}{cZ+k} \qquad 6)$$

where both c and k change at each inbuilding, the values of c as determined from Fig. 3 being

$$c = .35 \quad \text{up to } Z = 18$$
$$= .23 \quad Z = 18 \text{ to } 55$$
$$= .115 \quad 55 \text{ to } 87$$

or

$$c = \tau\rho \text{ where } \rho = .115 \text{ and } \tau = 1, 2, \text{ or } 3.$$

We are thus able to give a perfectly general expression for $\frac{\gamma}{n^2}$ in terms of the atomic number and constants peculiar to certain stages of development. This applies only where within the atom body there is a temporary completeness of group, and is the first step toward understanding the conditions which determine temporary completeness. Bohr has pointed out that final completeness is attained where the number in a given group is $2n$, but he offers no explanation of temporary completeness. We have shown that three definite conditions (i.e. $\tau = 1$, 2, and 3 respectively) successively determine completeness and that the transition from one to another is sudden and takes place whenever saturation is reached. In the saturated conditions $\frac{\gamma}{n^2}$ is simply proportional to the atomic number plus a constant i.e. from Fig. 5,

$$m = C\frac{1}{Z} + K \qquad C = 1.84 \qquad K = .03 \qquad 7)$$

so

$$\frac{Y}{n^2} = C + KZ = 1.84 + .03Z \qquad 8)$$

This equation defines the saturated condition reached before the formation of each new circular group.

INNER AND OUTER SCREENING

The total screening effect γ arises from two things—first, the decrease in actual force due to the repulsion of the electrons within the orbit; and second, the change in energy of the electrons lying outside the orbit, during transition, as mentioned at the beginning of the article. The former, Bohr and Coster (loc. cit.) represent by α and the latter by βn^2.

$$\gamma = \alpha + \beta n^2 \text{ or } \frac{\gamma}{n^2} = \frac{\alpha}{n^2} + \beta$$

Unfortunately there is no way to obtain direct information as to the value of either one. Bohr has shown that, on the basis of the older theory where the new groups of electrons added to the atom body were considered to form shells outside of those already present, the value of β would change by unity, for each added shell (each shell including all the orbital groups of the same total quantum number). From the standpoint of the new theory of interpenetrating orbits, whenever a new group is added, part of the electronic charge will go to

increase the charge lying inside of an established orbit and part will lie outside. That part lying inside will increase α while that outside will increase βn^2. From the table in paragraph 6 we see that $\frac{\gamma}{n^2}$ for the "K ring" increases by unity for each added "shell." From this Bohr

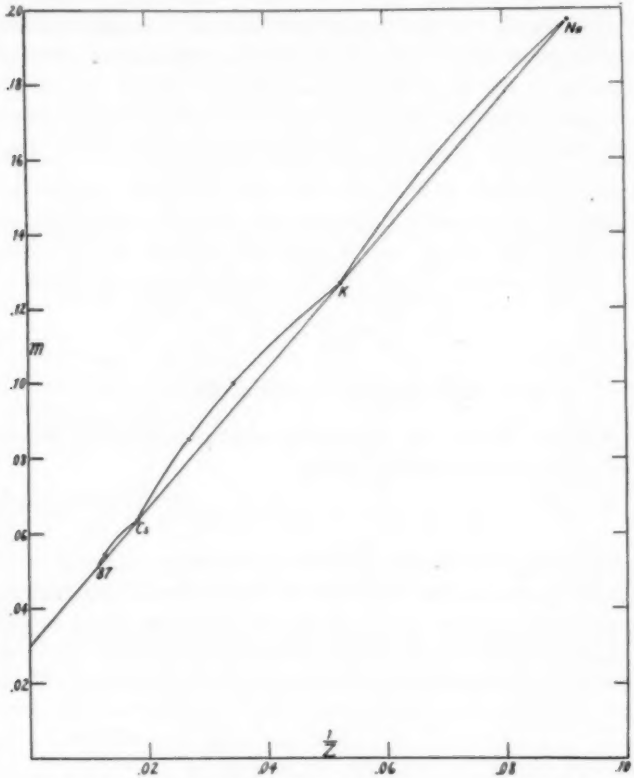

FIG. 5. *Linear Relation of Slopes for the "Saturated Conditions."*

concludes that $\frac{\alpha}{n^2}$ remains constant since β was expected to change by unity. The assumption which we shall make that both $\frac{\alpha}{n^2}$ and β will change, seems more tenable from the stand point of the new theory. Nor is it surprising that the total screening effect should change by unity just as expected from the older theory, for ours is simply a re-

division between α and βn^2 which would not be expected to affect the whole.

If now we examine the method of plotting which leads indirectly to the above equations, we find two factors which combine to lessen the accuracy near the nucleus of the expression for $\frac{\gamma}{n^2}$ namely (1) condensation of scale, and (2) the fact that γ is small compared to Z_a. In fact our value proves to be a weighted average where the conditions near the outer part of the atom body have been given very heavy weight. It amounts therefore to dealing with a condition in each atom where a small and equal amount of charge in each case lies outside. Under such conditions we can assume that $\beta = \epsilon$ (a constant), as it enters into our equations. Then by equation 8

$$\frac{\alpha}{n^2} = 1.84 + .03Z - \epsilon \qquad 9)$$

If β be small our curves suggest the assumption that $\frac{\alpha}{n^2}$ be a constant for a given atom (where there is only completed orbital groups). From equation 8) we can compute values of $\frac{\alpha}{n^2} + \epsilon$ for any atom satisfying the latter condition. If we subtract these values from those of $\frac{\gamma}{n^2}$ directly determined as in the table mentioned above, the residuals may be considered as values of $\beta - \epsilon$.

Table of Residuals $\beta - \epsilon$

	K	L	M	N
	$\Delta\beta_k$	$\Delta\beta_1$ $\Delta\beta_1$	$\Delta\beta_2$ $\Delta\beta_m$	$\Delta\beta_2$ $\Delta\beta_n$
Ar (18)	.3	.3 0		
Kr (36)	.6	.3 .3	.3 0	
X (54)	1.2	.5 .7	.5 .2	
Nt (86)	1.1–1.6	.8 1.1	.8 .4	.4 0*

* Wentzel's value

Here it is seen that, as would be expected, the residual is zero for the outmost orbit where $\beta = \epsilon$. The changes in β may be directly correlated with the number τ, which is characteristic of the charge distribution in

certain stages of the atomic development. The change in β from one circular orbital group to another is as follows:

$\Delta\beta_g = .3$ for Ar and Kr.
$\Delta\beta_g = .5$ for X.
$\Delta\beta_g = .6$ for Nt.

In passing from Ar to X and from X to Nt there is a change from one saturated condition to another. Here

$\Delta(\Delta\beta_g) = .2$ in the first range where $\tau = 2$
$\Delta(\Delta\beta_g) = .1$ in the first range where $\tau = 1$

Again if we take the change in the value of β for the K orbit as we pass from one saturated condition to another.

$\Delta\beta_k = .9$ in the first range.
$\Delta\beta_k = .4$ in the second range.

Then

$$\frac{\Delta\beta_k}{\Delta Z} = .025 \text{ in the first range where } \tau = 2$$
$$= .0125 \text{ in the second range where } \tau = 1.$$

From the standpoint of the new Bohr theory this is what is to be expected. When a new group is added, it produces a change in both α and β, since part of the effect is to increase the actual screening and part, due to that remaining outside, to increase the charge undergoing energy change. It is therefore to be expected that the change in β for an added orbital group should be less than the value of unity suggested by Bohr from the older theory, so that the values .3 to .6 seem reasonable. When the characteristic average distribution of charge in the atom body changes, as indicated by the change in τ, that is, where there is a condition of closer packing of which τ may be taken as the measure, the effect of each group will of course be different. The new group does not simply add another $\Delta\beta$ but changes as well, each existing $\Delta\beta_g$, establishing a new charge distribution.

In going from Kr to X, the 18 added electrons in eccentric orbits outside the 3_3 orbit produce an increase in β for 3_3 of .2 so that the value of ϵ due in the earlier elements to 8 electrons in eccentric orbits should be about .1 and certainly not over .15. Hence we can write:

$$\frac{a}{n^2} = 1.74 \pm .04 + .03Z$$

for elements in the saturated states. If the charge in the atom body is uniformly graded, remembering that

$$n^2 = Z_e a/a_0$$

we obtain

$$\frac{a}{Z_e} = (1.74 \pm .04 + .03Z)\frac{a}{a_0} = \cdot$$

This expression gives the simple type of charge distribution which is approached by atoms in the saturated condition.[5]

CONCLUSIONS

1. The distribution of apparent force in the atom body may be characterized by a simple number (slope m).

2. From the variation of this number with the atomic number, a second number τ is obtained which is constant in the ranges of uniform development.

3. The number τ changes by unity whenever a saturated condition is reached, which in turn is followed by the formation of a circular orbital group.

4. (a) The rate of change of the quantum defect for the virtual orbits is constant where τ is constant, (b) changes where τ changes; (c) the ratio of the values before and after change is equal to the ratio of the values of τ before and after.

5. This direct correlation of quantum defect with force field gives strong support to the quantum assignment as proposed in a previous article by Brackett and Birge upon which the determination of the quantum defect depends.

6. A separation of inner and outer screening effect is proposed which is compatible with the theory of interpenetrating orbits. This makes possible a determination of the charge distribution in the saturated conditions.

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UNIVERSITY OF CALIFORNIA,
BERKELEY, CALIF.,
APRIL, 1924.

[5] Since the presentation of this paper, the author has received an article by D. R. Hartee (Proc. Camb. Phil. Soc. XXI, VI.) in which the problem of the force fields has been approached in a somewhat similar manner. In order to obtain more information in regard to the points lying between these given by the circular orbits, data from the eccentric orbits have been used. This necessarily introduces further assumptions about which there is considerable uncertainty. The curves in his article show the resemblance to shifted equilateral hyperbolas (except for departures introduced by using the eccentric orbits) which is mentioned in this paper. He has however not plotted the reciprocals of Z_a and hence has not obtained the relations given in this paper.

Internal Photoelectric Effect of Rocksalt Crystals.—The conductance of perfect rocksalt crystals is not affected by ultraviolet or visible light; but crystals which have mechanical or chemical irregularities are photosensitive, and an insensitive crystal may be made sensitive by irradiating it with x-rays (which may produce a visible coloration). The experiments were performed according to the prescriptions of Gudden and Pohl; that is, the voltages and irradiations were applied only for brief time-intervals (a few seconds or fractions of a second), the effect measured with an electrometer shunted across the crystal, and the current in the first infinitesimal time-interval after the beginning of irradiation was computed by extrapolation. This initial current is strictly proportional to field strength over a surprisingly wide range—up to 50000 volts/cm—and shows no sign of impending saturation; it is likewise proportional to light-intensity over the range from .0003 to .005 cal/sec cm^2. However, the current diminishes steadily and rapidly from the first instant of irradiation onwards; there is some reason to believe that the field within the crystal is reduced by the steadily growing polarization, i.e., that there is something analogous to a "demagnetizing" effect. If only a part of the volume of the crystal is irradiated, the current has only a fraction of the value corresponding to complete illumination—the same fraction if the light is faint, a smaller one if it is bright. It is not clear whether the charges liberated by the light are free to move all the way to the edges of the crystal, or only a comparatively slight distance. If one electron is liberated by each absorbed quantum (which may or may not be true) the displacement of the charges is probably of the order 10^{-3} mm. [J. Bingel; ZS. f. Phys. *21*, pp. 229-241; 1924.]

K. K. DARROW

Near infrared Absorption spectra of some organic Liquids.—Measurements made with a high dispersion spectrometer, a thermopile and an automatic recording device, over the wave length range 0.8 to 2.1μ, on thirty-one organic liquids each containing both carbon and hydrogen atoms. Three absorption maxima at nearly constant wave lengths (1.7, 1.37, 1.17μ) were observed in all the 31 spectra, and two others near 1.02 and 0.9μ. These are ascribed to the CH combination present in all the molecules, and they may all be harmonics of a fundamental at 6.9μ observed together with a first harmonic at 3.4μ by Coblentz. Some of the bands are double, but this cannot be due to rotations, for the components of such doublets are farther apart for heavier molecules than for lighter ones. [J. W. Ellis, (Univ. of California) Phys. Rev., *23*, pp. 48-62; 1924.]

K. K. DARROW

PHOTOELECTRIC THRESHOLDS OF ELEMENTS UNDER ORDINARY CONDITIONS

By Richard Hamer

ABSTRACT

Photoelectric thresholds for sandpapered commercial samples of fifteen metals and graphite. Light from a carbon arc, sometimes with cored carbons, after dispersion by a mirror monochromator with a fluorite prism, was focussed through a fluorite window on the slit of the evacuated photoelectric cell containing a strip of the element being studied connected to a sensitive string electrometer, and the saturation photocurrent as a function of wave-length was determined. Scattered light was reduced to a minimum by the use of screens, and the effectiveness of these was tested by introducing filters. This method eliminates many of the usual difficulties. Commercially pure samples were tested with surfaces cleaned by sandpapering only. The fairly great intensity of the arc even to 2600 A gave reasonably large photocurrents as soon as the threshold wave-length was passed and enabled it to be determined from within ±30 A at 2600 A to ±100 A at 3600 A, corresponding to from ±.06 to ±.10 equivalent volts, as determined by the formula $V=h\nu_0/e$. The results in Angströms for Ag, Al, Bi, C (graphite), Ca, Cd, Cu, Fe, Ni, Pb, Pt, Se, Sn, Th, W and Zn are respectively: 3390, 3590, 2980, 2615, 4000, 3130, 2665, 2870, 3050, 2980, 2780, 2670, 3180, 3460, 2615, and 3420. These results are compared with those of others.

This research was undertaken with a view to determining directly the limiting frequencies for the various elementary metals under ordinary conditions. The principle of the method adopted was to use a source giving an approximately continuous spectrum extending over the experimental range of the limiting frequency for an element and to determine the threshold value by plotting the last values of the diminishing saturation photocurrents against the increasing wave-length of the incident light, or simply to take the average of the various wave-length limits observed in different determinations.

No special preparation of the metallic surfaces was made save that of rubbing with sandpaper, blowing with dry air, and placing them in the photoelectric cell which was immediately evacuated to a pressure of about 0.006 mm of mercury.

The Apparatus

The apparatus (see Fig. 1) was designed to cause as little absorption of the ultraviolet light as possible. It consisted of a modified Littrow type of spectroscope used with a concave mirror in the place of lenses. The light from the source Y entered the slit at S and was directed onto the face of the fluorite prism F by the concave mirror B, formed by cathodic spluttering of platinum on a glass concave lens. The prism was mounted in the box X so that the center of the face nearest the concave mirror was on the rigidly fixed axis of rotation of the whole box X, whose movement was controlled by a double tangent screw motion shown, in part, at T. A projecting rod A rigidly attached to the small central table supporting the prism, carried a guide G to which were pivoted two arms M and R, one fastened to the box carrying the

slit S and mirror B by means of a pivot at a post W, the other pivoted to the spectrometer arm I. Thus once a setting of C was made on a line for maximum deviation, rotation of the tangent screw caused other lines to appear, which in turn traversed the prism at minimum deviation. Since a mirror was used in place of lenses, the focusing of one line on the slit C was sufficient for all. In use it was found best to fix the spectrometer arm I rigidly to the table and move the whole box with slit and mirror by rotating the tangent screw. This necessitated attaching the source of light firmly to the box, which was managed by one or other of the two attached arms E and F. The rotation of the

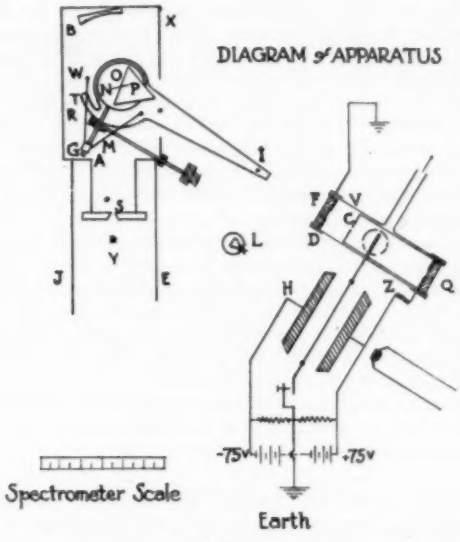

FIG. 1

prism was observed and measured by the reflection of a straight filament L as source from a plano-concave cylindrical glass lens N whose concave surface had been covered with a platinum film by cathodic spluttering and which was placed above on the axis of rotation. This gave an excellent line image on a ruled meter scale 153 cm away, visible even in daylight. In spite of a considerable backlash, a setting could be made to a fifth of a millimeter, which corresponded to an angle of 0.45 minutes of arc.

The photoelectric cell Z consisted of a brass tube closed at one end by a slit about 0.7 mm wide at C and at the other end by the quartz

window Q. The fluorite window F allowed the light of the desired wave-length from the prism to be focused sharply at C inside the exhausted glass tube V. To assist in correct focusing, the slit end C of the cell was coated with luminous zinc sulphide. The whole tube V could be rotated around a ground glass joint with a mercury seal placed below it on the exhaust tube.

The metal to be tested was in the form of a flat strip about 4x18 mm suspended from a brass rod sealed in glass with sealing wax and passing through a mercury sealed ground glass joint above. An area about 4x10 mm was exposed to the radiation by suspending it through the top of the brass photoelectric cell. By rotating this strip edgewise and by the use of a lens at Q, or by suspending a fluorescent screen in place of the metal strip, it was possible to set very exactly on some known line of the Hg arc spectrum.

The photoelectric cell described above was used in the case of Zn, Ag, Pt and Al. For the remainder, a modified and larger cell was used so situated that the monochromatic light could be focused just outside its quartz window. A much larger area was exposed and a slit about 20 mm long was used to increase the current.

The metallic strip was connected by the brass rod to a string electrometer with a sensitivity from 50 to 100 divisions per volt but with a capacity of only about 25 cm. This is represented diagrammatically at H. The knife edges were charged to ± 75 volts with respect to the earth.

The capacity was kept as low as possible by having the electrometer close, and, of course, it was necessary to shield everything from electrostatic induction. The Faraday chamber was connected to the knife edge positively charged. The cell was about 8 cm in length in a glass tube about 12 cm long and its diameter was about 2.5 cm. The slit of the cell was about 41.5 cm from O, the axis of rotation, which was 12 cm from the mirror, which in turn was 31 cm from the slit. The ultraviolet light, therefore, had to travel about 84.5 cm in air after passing through the slit.

Owing to the considerable amount of scattered light of all frequencies, it was desirable in some cases to use filters to absorb as far as possible the diffuse light of higher frequency than the selected one.

CALIBRATION

On first setting up the photoelectric cell, it was necessary to be sure that the line on the slit C was due to the wave-length at minimum

deviation, so that the dispersion curve for fluorite could be used in the identification of the lines. Therefore a reading microscope sliding in a long tube supported along the arm *I* but directed towards the middle line of the nearest face of the prism, was used to set on the sodium D lines and the visible mercury lines. Corresponding prism angles were noted. The tube and microscope were now removed because the tube reflected light internally, and the photoelectric cell was then arranged to catch the particular wave-lengths at the same prism angles as denoted by the scale.

The lines were next visually identified by means of a zinc sulphide screen and comparison with spectrograms of the mercury arc spectrum. However, in order to make absolutely certain of the effective location of monochromatic radiation on the metallic strip, a series of observations of the photoelectric current were taken for each metal at every 2 mm of the scale, namely at about every 4.5 minutes of arc. This enabled an accurate calibration to be made and safeguarded against accidental displacement of the cell while a transfer of elements was taking place, besides assisting in giving a rough determination of the threshold value.

With this experimental arrangement a series of measurements of the relative photoelectric currents at different wave-lengths was made. If there were no effects due to scattered light and if the slit were infinitely narrow, the current would begin to rise rapidly at the critical wave-length. The effect of both of these is to make a rounded, instead of a sharp, break in the current-wave-length curve. By the use of thin glass filters the effect of shorter wave-lengths was reduced.

The slit was 0.7 mm wide, covering a range of from 36 to 100 angströms, approximately. It is assumed that the two points of rapid inflection in the curve (Fig. 2) indicate the points at which the critical wave-length began and ceased to fall upon the slit, and the effective wave-length was taken as lying between these two points. The average was taken, generally.

The line λ2536 of the mercury arc was used as a point of reference, and the critical wave-lengths determined in terms of distance from this point, with the help of the calibration curve.

OBSERVATION AND RESULTS

Zinc. The results of fourteen series of observations some without and some with a thin glass filter, and with the mercury arc, the solid carbon arc, and the core-soaked carbon arc gave a final average λ3382. Carbons

with soft cores were soaked in a concentrated solution of iron, nickel and uranium sulfates.

The solid carbons probably gave more accurate results since they would not be so likely to produce much ultraviolet light below the threshold value for zinc and so would make the use of filters less necessary. Thus larger photocurrents were secured and hence more reliance was placed on these.

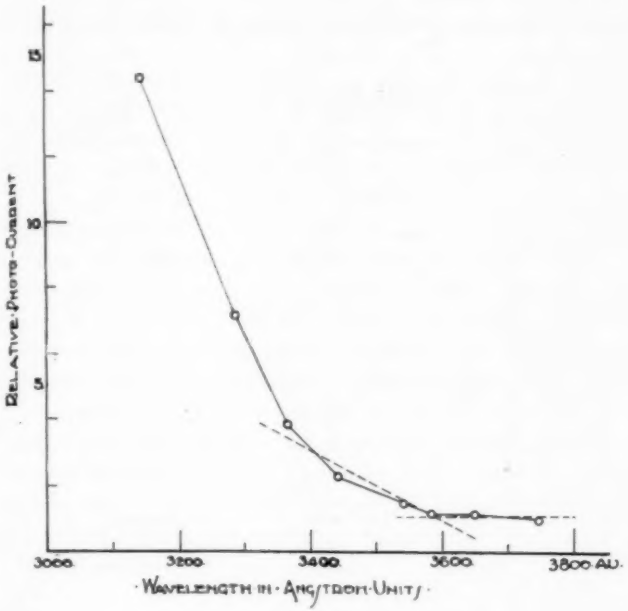

FIG. 2

In detail they are as follows, the readings indicating the limits between which the curve changed its gradient:—

Solid Carbon Arc

No Filter	Thick glass filter
λ3380 to λ3406	λ3406 to λ3480
λ3406 to λ3480	

The average of these three is λ3426 A ranging from λ3365 to λ3499 if we consider 2 mm as the experimental error due to width of slit, impurity of spectrum, uncertainty of exact end point due to the readings being of long duration usually, and the effect of zero drift. It would

appear that the limiting value for zinc could have been obtained directly with the carbon arc, but greater confidence was placed in the results when they were tested with filters that absorbed much of the shorter wave-length radiation.

Aluminum. The readings observed on aluminum with the carbon arc (soft cored but not soaked) using a glass filter arc are plotted in the typical curve, Fig. 2, which indicates the threshold value for aluminum to be about $\lambda 3596 \pm 100$.

The following table summarizes the results of the determinations for other elements.

TABLE 1.—PHOTOELECTRIC THRESHOLDS

Element	Hamer	Hughes	Richardson and Compton	Hennings	Other Observers	Equivalent Volts
Graphite	2615 ± 30					4.72 ± .06
Al	3595 ± 100		4770	3342	3650 Hennings	3.43 ± .10
Ca	4000 ± 200	3700				3.08 ± .16
Fe	2870 ± 40			2804–3132		4.30 ± .06
Ni	3050 ± 50					4.04 ± .07
Cu	2665 ± 30		3000	2804–3122		4.63 ± .05
Zn	3425 ± 75	3016	3760	3132–3342		3.60 ± .08
Se	2670 ± 35	2200				4.62 ± .06
Ag	3390 ± 60				3250 Stuhlmann	3.64 ± .07
Cd	3130 ± 50	3140			3130–3650 Pringsheim and Pohl	3.94 ± .06
Sn	3185 ± 55		3620	2804–3132		3.87 ± .07
W	2615 ± 30				2300 Hagenow	4.72 ± .06
Pt	2780 ± 35		2800		(2840 Stuhlmann (2570 Koppius	4.44 ± .06
Th	3465 ± 75					3.56 ± .08
Pb	2980 ± 50	3115				4.14 ± .07
Bi	2980 ± 50	3233	3300		3130-3650 Pringsheim and Pohl	4.14 ± .07

The surfaces were generally prepared from commercially pure material in the form of strips. These were vigorously rubbed with fresh sandpaper while supported on clean filter paper until the surface crust was rubbed away and a clean surface of the element exposed. To remove loose particles, the surfaces were first wiped off with fresh filter paper and then subjected to a current of dry air. As quickly as possible the strip was suspended in the cell and within a few minutes the cell was exhausted of air by a Gaede mercury pump.

In certain cases where the strips were replaced by others no marked difference in the limiting frequency was noticed. Sometimes a strip was removed and submitted to another similar cleaning treatment in case the pressure for some reason or other had been much increased. Therefore it may be stated that the main precaution that was taken, and intentionally so, was to remove and avoid the formation of a badly contaminated greasy or oxide film on the surface.

The table gives other determinations for comparison, some taken with much greater care in preparation and treatment. It is surprising that the values agree as closely as they do.

The last column gives the equivalent value in volts of the energy required to release the respective photoelectrons from the surface under the conditions of the experiment. This is calculated by the use of the quantum relation, $Ve = h\nu_0 = hc\lambda_0$.

This work was more or less preliminary to a more precise determination of the threshold values with more elaborate apparatus whereby investigations regarding possible shifts of these values under special conditions could be accurately determined, such as better vacuum conditions, outgassing, and special preparation of the surfaces of the substances to be studied. The results presented here simply apply to the elements under ordinary conditions and it is hoped that they may be of real assistance to anyone designing photoelectric cells or considering photoelectric properties under ordinary conditions.

My thanks are indeed due to Professor E. P. Lewis for his kindly interest and suggestions throughout the course of this research.

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Reflecting Altimeter with Lamp and Scale.—The scale of the aneroid altimeter is often not uniformly divided. To obviate this defect a very delicate cam system is sometimes used which does not always function properly, with the result that irregularities ensue.

By using an optical system of reading, these difficulties are said to be avoided, and at the same time provision made for the adjustment of the zero of the scale.

The aneroid capsule acts directly on a small mirror set in jewelled bearings, and produces rotation of the mirror. A small lamp and projection system throw a beam of light on the mirror which reflects it to a translucent strip where the light is converged to a line or spot image. This screen is not on an arc concentric with the axis of rotation of the mirror but is so bent that equal changes in altitude produce equal displacements along the screen, in much the same manner that a

galvanometer scale may be curved to give a strictly linear calibration. The scale of altitude is placed on another strip alongside the first and is adjustable by means of a knob to eliminate zero error. The device is arranged for convenient use when mounted on the instrument board of an airplane. [Revue d'Optique, *2*, No. 3, pp. 116-119; 1923.]

G. W. MOFFITT

Report on Radiation and the Quantum Theory.—By J. H. Jeans. 2nd Edition 84 pp. Fleetway Press, London, 1924.

This report is a revision and enlargement of the first edition; it does not claim to be an abstract of present-day quantum theory, but is concerned mainly with the fundamental concepts and the brilliant early achievements of the theory. We shall not discuss the numerous errors which are made in the discussion of purely experimental matters, such as the paragraph on the production of the Balmer series in the laboratory, p. 40; or the statements about absorption spectra, in which no attention is paid to selection-principles; or the indiscriminating compilation of data on the fine structures of hydrogen lines; or the statement that Zeeman effect patterns have a wider spread than Stark effect patterns under customary laboratory conditions.

It will be of more profit to point out the best features of the report. The first two chapters deal with the need for the quantum theory, and the problem of black radiation as treated by the classical theory. In the third chapter Jean's familiar proof of Planck's law is given, and the inconsistencies of Planck's original demonstration are discussed. Then comes a variation of Einstein's proof which shows very clearly that Wien's displacement law makes the relation $\epsilon = h\nu$ inevitable. The author points out carefully that Einstein's hypothesis is: "The interchange of energy between matter and radiation is always by single complete quanta of radiant energy," which does not by any means imply that radiant energy or the energy of Planck resonators can exist only as complete quanta.

The necessity for Planck's law is then discussed from points of view due to Jeans and to Poincaré. The fourth, fifth, and sixth chapters deal with the spectra of hydrogenic atoms, the specific heats of solids and the photoelectric effect, respectively. Chapter vii deals with the quantum conditions for systems in which separation of variables can be accomplished. The paragraph on adiabatic invariants would have been better if it had followed more closely the longer treatment in Jeans' book on kinetic gas theory. The chapter closes with some simple illustrations of the correspondence principle.

The last chapter is concerned with Jeans' own views on modifications of classical dynamics. Much of this is familiar from his Guthrie lecture.

Workers in atomic physics will need to read only sections 23 and 65 and chapter viii of this report; but the entire work is probably the most readable and comprehensive introduction to the quantum theory in English.

ARTHUR E. RUARK

INSTRUMENT SECTION

PRECISION X-RAY SPECTROMETERS

By J. C. Hudson

In 1913 Laue predicted that x-rays should be diffracted when passed through a crystal. Friederick and Knipping verified this prediction experimentally and a short time later W. H. Bragg showed that the x-ray beam is broken up into a spectrum by reflection from the atomic planes of a crystal as in the case of the reflection grating for visible light. This diffraction by reflection from the atomic planes is the principle made use of in x-ray spectrometers. A classification of the instruments may be made on the basis of the method of determining the position of the spectral lines. One type of instrument makes use of the ionizing effect of the rays and the other of the effect on a photographic plate. In the former, the rays reflected from the crystal planes are directed into an ionization chamber, generally filled with a heavy gas, and the relative amount of ionization produced is determined by an electrometer or an electroscope. By plotting the ionization currents against angular displacements of the crystal the position of the line may be determined. This method was first used by W. H. and W. L. Bragg, and in the laboratory of Professor W. Duane it has attained a high degree of accuracy.

In the photographic method the reflected ray is received directly on a photographic plate as a slightly curved line and its position may be readily determined as will be explained later. The method has been highly developed, the work of Professor Siegbahn and his students at Lund being the most extensive and probably the most accurate.

Professors H. S. Uhler and C. D. Cooksey[1] of Yale have developed a precision x-ray spectrometer which gives very accurate results but which has been used comparatively little and will not be described in detail. In using it the reflected lines are received on a photographic plate in a given position, the x-ray beam having been collimated by two slits of equal width placed between the x-ray tube and the crystal. The photographic plate is then displaced in a straight line parallel

[1] Phys. Rev. *16*, p. 305; 1920.

to the line of collimation and the lines again taken. The angle of reflection at the crystal is determined by the distance apart of the spectral lines, taken on both sides of the direct image of the slits in the two positions, and the distance through which the photographic plate has been displaced.

The spectrometers discussed in this paper are those of Professor Siegbahn at Lund and that of Professor Duane of Harvard. In determining the wave-length Bragg's law is used, $n\lambda = 2d \sin\phi$, in which λ is the wave-length and ϕ the angle which the x-ray beam makes with the reflecting planes of the crystal when the crystal is placed at such an angle that the spectral line represented by λ is received on the photographic plate or in the ionization chamber. d represents the lattice constant or perpendicular distance between the reflecting planes of the crystal used. In the precision measurements d is regarded as known without error since no method has been devised of determining it with the high degree of accuracy which is possible in determining ϕ. If at some future time d is determined with greater accuracy it will be a simple matter to correct the values of the wave-lengths.

In his work at Lund Prof. Siegbahn has designed three accurate spectrographs for use over different regions of the spectrum. All are of the photographic type and differ essentially only in details of construction and in the method of measuring ϕ. The vacuum spectrograph is used for wave-lengths greater than 1.0A which would be largely absorbed by air and by the glass walls of the tube. Another is for use in the region of wave-lengths between .4 and 1.0A and the third for use with wave-lengths below .4A. In the determination of wave-lengths by the vacuum spectrograph use is made of the fact that an x-ray beam in the direction cd (Fig. 1) will be reflected in two directions in the first order if the crystal is placed in the correct positions. Thus in the figure with the crystal in position 1, a line is secured on a photographic plate placed at position AA'. Now if the crystal is turned through an angle equal to $180° + 2\phi$ and the plate-holder through an angle 4ϕ a second line will be secured which exactly coincides with the first, the first order spectrum being used in each case. Actually, the crystal need not be turned through exactly the correct angle since the instrument is so constructed that the accurate placing of the crystal is unnecessary, as explained in the next paragraph. The angle ϕ is the angle of reflection in Bragg's law and together with d serves to evaluate the wave-length.

The essential parts of the vacuum spectrograph[2] are shown in Fig. 2. The crystal and photographic plate-holder are carried in a cylindrical brass box which may be evacuated. Since in this spectrograph the angle of reflection, ϕ is measured by the position of the line on a photographic plate it is essential that the position of the plate-holder be read very accurately. It is also essential that the crystal and plate-holder turn about the same axis and that the slit and plate-holder be the same distance from this common axis of rotation, that is, that they lie on the circumference of a circle having the axis of rotation as the center. Bragg[3] has shown that only under this condition is it unnecessary to determine accurately the angular position of the crystal.

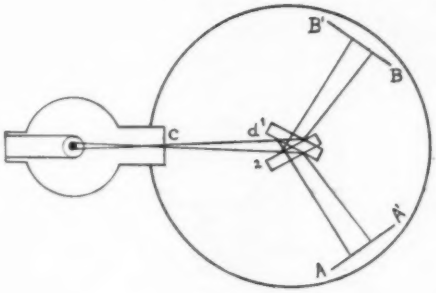

FIG. 1

As seen from the figure the crystal table is carried on a cone which extends through the bottom of the metal box and which carries on its lower end two cross bars at right angles and each carrying at either extremity a vernier. These verniers move over a scale graduated in degrees on which is read the position of the crystal. This scale is used only to place the crystal in approximately the correct position to reflect a given wave-length. The crystal table itself is fitted with a screw for adjusting the crystal about a horizontal axis in order that the reflecting planes may be made parallel to the axis of rotation and a micrometer screw and drum provides a means of moving the crystal in a horizontal plane. This latter combination may be read to one thousandth part of a millimeter.

[2] M. Siegbahn, Jahrbuch d. Rad. u. Elektronik *18*, p. 240; 1921.

Since the present paper was written Prof. Siegbahn has published an excellent book on x-ray spectroscopy entitled "Spektroskopie Der Röntgenstrahlen."

[3] W. H. and W. L. Bragg, X-rays and Crystal Structure.

Rotating about the same axis as does the cone carrying the crystal is another cone which carries the plate holder. Since the position of the plate-holder is desired with as great accuracy as possible a very accurate

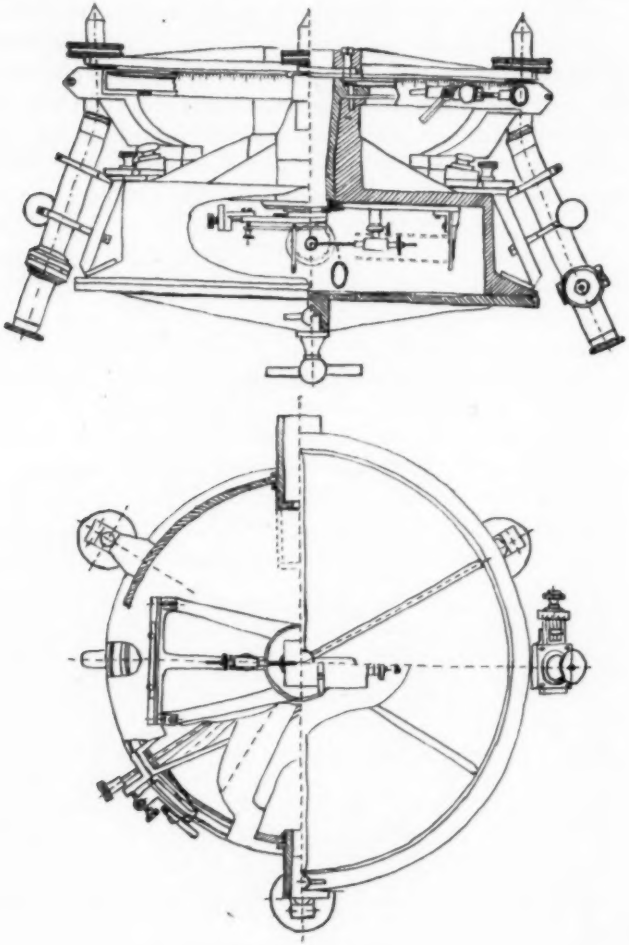

FIG. 2

circular scale is fastened to this latter cone and readings are taken by means of the two micrometer microscopes shown. In the latest spectrographs designed by Prof. Siegbahn the plate-holder may be moved

relative to the circular scale in order that by using different parts of the scale errors due to eccentricity may be reduced to a minimum. Mounted on the plate-holder in the figure may be seen a small ivory point, *O*, which is used in adjusting the crystal and will be mentioned again later. The photographic plate must lie in the plane tangent to the circle which passes through the slit and has as its center the axis of rotation, and for the precision measurements the spectral lines should lie not more than one or two millimeters from the point of tangency. A micrometer tangent screw serves to place the plate-holder in the desired position.

In the earlier spectrographs a slit-width of about 0.1 mm was used but in the later ones an adjustable slit is used which can be made as small as 0.025 mm. For studying the fine structure of the spectral lines it is essential that the slit be made as narrow as practicable. The slit used consists of the halves of a steel cylinder which has been cut along

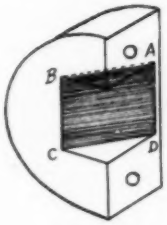

FIG. 3

its axis. The two halves are fastened together by screws. They are highly polished and as shown[4] in Fig. 3 a prismatic piece *ABCD* is cut out leaving a strip of the original surface about 1 mm wide at one end of the cylinder. This is again polished and by screwing the two halves together with no plates between them, slit-widths of about 0.025 to 0.030 mm are secured. The width may be increased by inserting thin sheets of metal between the two halves. The slit is fastened directly to the cylindrical box.

The x-ray tube used with the vacuum spectrograph is of the hot cathode type and is water cooled, the general form being shown[5] in Fig. 4. In order to secure as great intensity as possible the tube and spectrograph are joined directly together. The slit forms a part of the wall of the tube at a distance of about 25 mm from the target. The tube it-

[4] M. Siegbahn, ZS. f. Physik, *9*, p. 68; 1922.
[5] M. Siegbahn. ZS. f. Physik *9*, p. 75; 1922.

self is made of metal and earthed, the terminal carrying the high potential being insulated by means of a glass sheath. A narrow beam of rays is desirable in order to secure very fine lines on the photographic plate. At the same time, as great intensity as possible is desired. For this

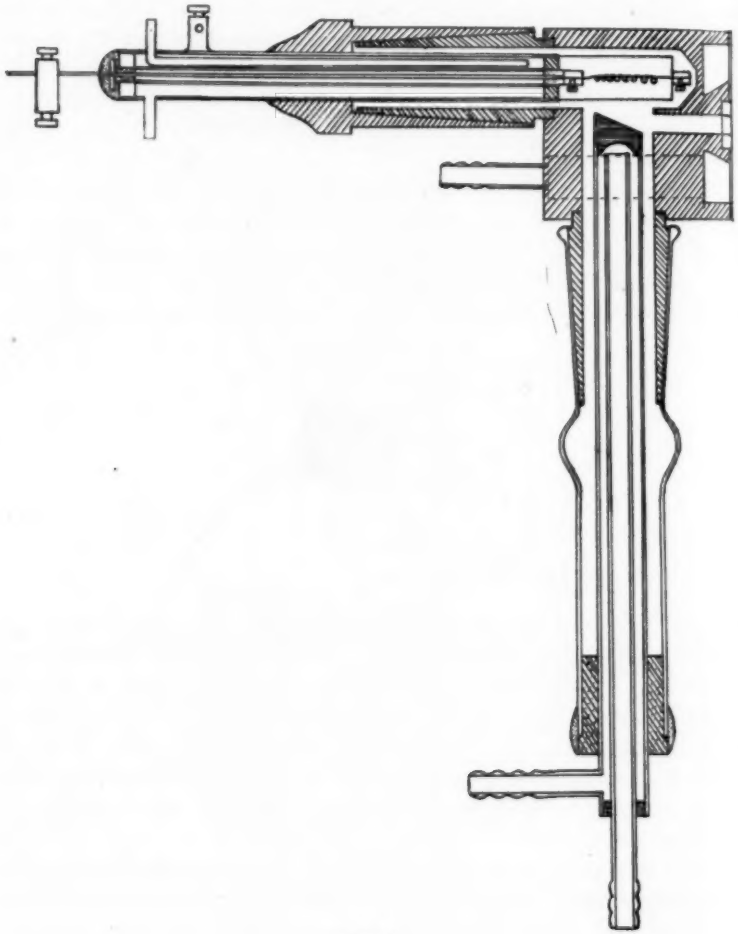

FIG. 4

reason the anticathode is placed almost horizontal and the cathode filament drawn out into a line parallel to the direction of the x-ray beam. The filament is surrounded by a sheet-iron cylinder with a slit

so that the source of the x-rays is a line less than 1 mm in width and extending the length of the target face. It is necessary for accurate measurement that the beam of rays be directed toward the center of the spectrograph and for this purpose the sheath carrying the cathode may be revolved, and is fitted with a scale for reading its position. The adjustment must be made anew each time a different filament is used. The vacuum in the x-ray tube is separated from that of the spectrograph by a very thin foil or by a thin sheet of goldbeater's skin together with a sheet of black or red tissue paper. This allows the spectrograph, in which a high vacuum is not necessary, to be connected to the secondary vacuum pump and also makes it less difficult to secure a good vacuum for the x-ray tube.

For a high degree of accuracy several important adjustments are necessary, the two most important being the placing of the crystal planes parallel to the axis of rotation of the crystal table and placing the reflecting planes on this axis. In order to make the first adjustment a small plane-parallel piece of glass is placed firmly against a vertical face of the crystal and a telescope and scale used to determine the correct position. A second determination is taken with the glass plate reversed, since it may not be accurately plane-parallel. The average of the two readings is considered the correct adjustment.

The most important adjustment of all is placing the reflecting planes of the crystal on the axis of rotation of the instrument. This, as will be seen later, is the source of greatest error. As an aid in this adjustment the small ivory point, *O*, (Fig. 2) is placed accurately on the axis. This is done by mounting a microscope parallel to the axis and after focusing it on the ivory point, turning the plate-holder (carrying the point) about the axis of rotation. In this way it can be placed very accurately. Now the crystal is placed on the crystal table and the horizontal micrometer screw and drum turned until the crystal just touches the ivory point. This instant may be determined by focusing the microscope on this point and its image in the crystal face and moving the crystal until the point and its image touch. The optical surface is used for the placing of the crystal and the question naturally arises as to whether the optical surface is parallel to the atomic planes, which are the reflecting planes of the crystal, and if this is not true, whether or not it affects the measurements. The wave-lengths measured by the vacuum spectrograph are comparatively long and easily absorbed and so the reflection from the crystal takes place at the outermost layers of atoms. Since the optical surface is placed on the axis of

rotation, if the atomic planes are parallel to this surface, lines taken first with the crystal at rest and then with it rotating slowly should be of practically the same breadth. If the atomic planes were not parallel to the optical surface the lines would be broader when taken with the crystal rotating since the reflecting planes would be displaced from the axis of rotation. Professor Siegbahn and his students have performed this experiment many times with different crystals and no noticeable broadening has resulted.

The focal spot on the target furnishes practically a point source for the rays and so the lines on the plate will be arcs of hyperbolas since they are the intersection of the plane of the photographic plate with the cone of rays from the source. In determining the angle of reflection two lines are secured for each determination, one on either side of the direct beam, and since they are slightly curved there will be a point of mini-

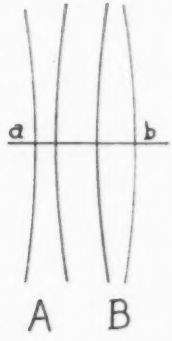

FIG. 5

mum or maximum distance between them. The lines will appear as in Fig. 5, *A* or *B*, depending on whether the determination is a "direct" or "crossed" one as explained below. In order to secure sufficiently accurate results in the precision determinations, the measurements of the lines must be made in the horizontal plane in which the focal spot of the anticathode lies. The line *ab* in Fig. 5 represents the intersection of this plane with the photographic plate. Professor Siegbahn[6] has shown that measurements of the distance between the lines must be made in a region about *ab* of approximately 1 mm width vertically, if an accuracy of .001 mm is desired, as is the case in his work. It is

[6] M. Siegbahn, ZS f. Physik, *9*, p. 68; 1922.

necessary, then, that all adjustments and measurements be made in the horizontal plane in which the focal spot lies.

In making a determination the crystal is set at approximately the correct position by means of the four verniers and the photographic plate is placed accurately at some definite position on the scale. An exposure is made and the crystal is turned through an angle of $180° + 2\phi$ where ϕ is the angle the direct beam makes with the reflecting planes of the crystal. The photographic plate is turned through an angle very nearly equal to 4ϕ and another exposure made. Thus two lines are secured on the photographic plate and the angular distance between them, together with the angle through which the plate itself has been turned, gives four times the angle of reflection. The angular distance between the lines is secured by measuring the distance between them and dividing by the radius of the spectrograph. For the precision measurements the distance between them is measured on the dividing engine to .001 mm. The radius is secured by measuring the perpendicular distance from the plate to the small ivory point on the axis by means of an adjustable rod.

Two types of determinations are possible. If the plate-holder is turned through an angle less than 4ϕ a "direct" determination is secured and in calculating 4ϕ the angle between the lines is added to the angle through which the plate is turned. If the angle through which the plate is turned is greater than 4ϕ we have a "crossed" determination and the angle between the lines is subtracted from the angle through which the plate turns. The lines in the "direct" and "crossed" determinations appear as in Fig. 5, *A* and *B* respectively. The following is a typical determination as taken from a paper by Hjalmar.[7]

Determination of Sn $L\alpha_1$ in second order.

ϕ is known to be about 28° 18′.

Zero position of plate-holder	236°25′	Zero position of crystal	268.7°
2ϕ	56°36′	ϕ	28.3°
1st position of plate-holder	292°61′ − 11′	1st position	297.0°
2nd position of plate-holder	179°49′ + 11′	2nd position	240.4°
Turning of plate-holder	112°50′		

The angle through which the plate is turned is too small so that we have a "direct" determination. The distance between the lines is found to be .949 mm and since the radius is 182.966 mm this corresponds to an angle of 17.83′. Then $\phi = \dfrac{112°\ 50' + 17.83'}{4}$.

[7] Hjalmar; Dissertation at Lund, 1923.

In case of weak lines, measurements are taken on one side of the zero only as shown in Fig. 6. A reference line is taken, frequently Cu $K\alpha_1$ being used, and the desired line measured with reference to it. In this case $\phi = \frac{D \pm \Delta\phi + 2\alpha}{2}$. D is the angular distance the plate-holder is moved between the taking of the reference line and the desired line in the position in the figure denoted by the dotted line. $\Delta\phi$ is the angle between the reference line and the line to be measured. 2α is twice the reflection angle of the reference line. The + or − sign is used when the line lies on the long or short wave-length side of the reference line. The distance between the lines for determining $\Delta\phi$ is measured in the case of weak lines by means of a millimeter scale and a magnifying glass, distances being estimated to tenth millimeters.

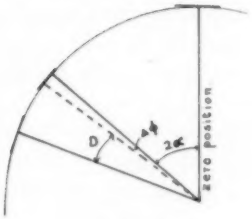

FIG. 6

In calculating the wave-length by Bragg's formula, $n\lambda = 2d \sin\phi$, all inaccuracies enter through the measurement of the angle ϕ since we assume d correct. Errors due to the eccentricity of the scale are very small in the latest vacuum spectrographs, Prof. Siegbahn estimating the error as in the neighborhood of 0.5″.

The penetration of the rays into the crystal has a tendency to remove the effective reflecting planes from the axis of rotation but with wave-lengths which are measured by the vacuum spectrograph the penetration is probably negligible. In the photographic spectrographs the breadth of the lines in comparison with the slit-width gives an approximate representation of the depth of penetration, and on this basis the thickness of the reflecting layer is less than .001 mm.

The inaccuracy in placing the reflecting planes on the axis of rotation is the source of largest error. The approximate amount of this error can be easily calculated. Suppose the reflecting planes are the distance δ from the axis of rotation, the adjustment having been made with the crystal surface parallel to the photographic plate. Then the line on the

plate is displaced a distance $2\delta \cos \phi$ where ϕ is the angle of reflection. If ϕ is small $\cos \phi$ may be set equal to 1 and the displacement of the line is 2δ. The error in placing the planes is about .001 mm and therefor the error in position of the line will be .002 mm. Since the errors in the positions of the two lines are additive we may expect a total error of .004 mm.

The distance between the lines is accurate to but .001 mm and so the total error in the distance is about .005 mm and if the radius is 185 mm this corresponds to an error of 5.6″. Combined with the eccentricity error of 0.5″ the error in the angle is a little more than 6″. Since this angle represents the four-fold variation of ϕ the error in ϕ is 1.5″. The percentage error varies with the magnitude of ϕ and for $\phi = 15°$ is approximately .003 per cent. For smaller angles it is proportionately greater. For the Cu $K\alpha_1$ line using the calcite spacing such that log $2d = 0.7823347$, Prof. Siegbahn secures the value $\lambda = 1537.302 \times 10^{-11}$ which he estimates as correct to .002 per cent.[8]

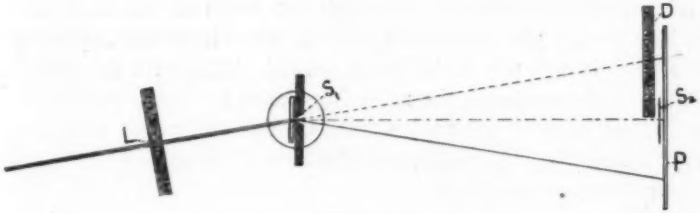

FIG. 7

With very hard x-rays care must be taken that penetration of the rays into the crystal does not affect the accuracy of the determination. For this reason Prof. Siegbahn has employed a special spectrograph for the measurement of very short wave-lengths.

The essential parts of the instrument are shown in Fig. 7. The slit S_1 and plate P form opposite sides of a pyramidal-shaped cast iron box lined with lead 10 mm thick. The top of the box is removable for convenience in adjustment. The box rotates about a perpendicular axis through slit S_1 and carries a vernier which is read on a circular scale carried on the bed-plate of the instrument. Slit S_1 is of small angular opening and has gold edges. Two thick lead plates, one of which is shown at D, are used to protect the plate from the direct beam during the determination of the spectral lines. L is a slit fixed to the bed-

[8] M. Siegbahn, P. M. *38*, p. 647; 1919.

plate and serves to direct the beam from the x-ray tube. S_2 is an auxiliary slit for the purpose of securing the position of the zero. It is so placed that the perpendicular from S_1 to the photographic plate passes through it. The zero position in each spectrogram is determined by placing L, S_1 and S_2 in the same straight line and taking an exposure of about 20 seconds. After the adjustment of S_2 the crystal is placed directly on S_1 with its inner atomic planes *parallel* to the vertical plane through S_1 and S_2. This adjustment may be checked by taking the same line on both sides of the zero position. If the adjustment is perfect the line will lie the same distance from the zero line in the two cases. In general this will not be true and a small correction must be made to the values of the angles found. Since the zero line should lie mid-way between the two spectral lines, the correct position may be found by measurements of the distances between the three lines. After this correction is once found for a particular set of determinations the plate need be exposed only on one side of the zero to secure as great accuracy as the double exposure without the zero line. The zero line is taken each time and since the angles are small the distance between the lines on the plate divided by the distance from slit to plate gives the angle through which the box is turned. This, with the correction mentioned above due to the reflecting planes not being parallel to the line joining the slits, determines twice the reflection angle to be used in Bragg's formula. The slit-plate distance is about 500 mm and is measured very accurately.

The relative accuracy of measurement with this instrument is less than with the vacuum spectrograph. While the penetration of rays into the crystal and the failure of the reflecting planes to coincide with the axis of rotation are practically eliminated as sources of error, the fact that the zero position must be corrected as explained above makes the actual error in determining the total angle fully as great as in the determinations by means of the vacuum spectrograph. The error in the angle is in the neighborhood of $6''$ and since this represents 2ϕ the error in ϕ is about $3''$, which represents for a value of ϕ of $2°$, an error of 0.04 per cent.

The two spectrographs described above have been supplemented by a third for the region between .4 and 1.0 A. The instrument used for hard rays is not suitable since the angles in this middle region are too large for accurate measurement by the method used, while the vacuum spectrograph is unsuitable due to the penetration of the rays into the crystal. The new instrument is a combination of both the others in that

the angle of turning of the crystal is read from a precision circular scale and the slit is between the crystal and the plate as in the instrument for hard rays.

A schematic diagram of the instrument is shown[9] in Fig. 8. A precision scale is fastened to the crystal table and is read by two microscopes which are securely fastened to the large base which carries the photographic plate. On this same base, between the crystal and plate-holder and close behind the former, is the slit. A tangent screw serves to turn the circular scale relative to the plate-holder while both crystal and plate-holder may be turned about a vertical axis through the center of the crystal table. Two rather broad slits are fixed between the crystal and x-ray tube and serve to direct the beam of rays toward the axis of rotation of the crystal.

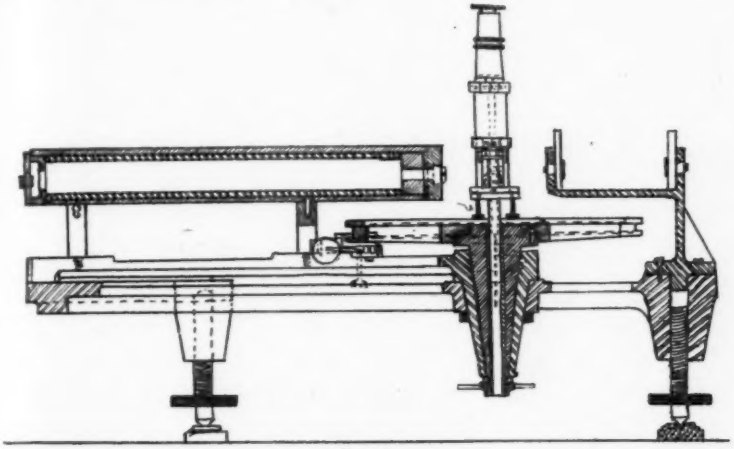

FIG. 8

The slit is so arranged that a line from the axis perpendicular to the photographic plate passes through it and, furthermore, is made quite narrow since it serves to define the reflected beam and a very narrow line on the photographic plate is essential for securing accurate measurements. The crystal is placed with its reflecting planes on the axis of rotation and the instrument is ready for use. The determinations are very similar to those with the vacuum spectrograph except that here the two-fold angle of reflection is determined by means of the circular scale fastened to the crystal table together with the small correction secured from the measurement of the distance between the lines on the

[9] M. Siegbahn, Jahrbuch d. Rad. u. Elek. *18*. p. 240; 1921.

plate. Suppose the plate-holder and crystal have been turned about the axis and adjusted relative to each other so that a reflected beam is received on the plate. The angle between this direction and the zero position is approximately 2ϕ where ϕ is the angle of reflection. After the exposure of this line the photographic plate together with the crystal is revolved through an angle of about 4ϕ. The reading of the crystal table having been made by means of the microscopes the crystal is turned through an angle nearly equal to $180° - 2\phi$ and a second exposure made. The lines on the plate will be very nearly superposed, and by measuring the distance between them and dividing by the distance from axis to plate, the correction for the angle is secured. This small angle represents twice the correction to be applied to the angle determined by the difference in the readings of the two positions of the crystal and if we designate it by a and the angle determined by the readings on the scale by D, we have $2\phi = D \pm \frac{a}{2}$ or $\phi = \frac{D}{2} \pm \frac{a}{4}$.

The determinations by means of this instrument are not so accurate as those in which the vacuum spectrograph is used since in the latter case the four-fold reflection angle is measured. Assuming that the measurement of radius, distance, etc. are as accurate here as in the case of the vacuum instrument, we would expect a probable error of about twice as much, or in the neighborhood of 3″ for any measurement. The probable error as calculated from a series of measurements with this instrument gives a value of 3.1″ which for angles near 15° gives an error of approximately .006 per cent.

The actual errors in the measurements of the angles by means of the vacuum spectrograph and the one just described are exceedingly small but the values of λ cannot be regarded as determining real wave-lengths to such a degree of accuracy as is secured in the angular measurements. The error introduced into λ due to inaccuracies of measurement of ϕ is less than that caused by refraction of the x-ray beam in the crystal. When ϕ is in the neighborhood of 5° the value of λ when determined from first order reflection may differ as much as .05 per cent from λ as determined from second order reflection. As yet no satisfactory method of correcting for refraction has been devised, so that while the probable error in any measurement is of the order given the values tabulated cannot represent actual wave-lengths to the same percentage accuracy.

However, the three instruments described here do make possible accurate measurements over a large region of the x-ray spectrum and

the vacuum spectrograph, especially, as improved by Siegbahn, marks a real advance in x-ray spectroscopy since now the very soft *M* and *N* series of wave-lengths are being determined.

The ionization spectrometer as developed by Prof. Duane and his students at Harvard has several distinct advantages over the spectrometers which make use of the photographic effect of the x-rays. By proper arrangement of the slits, source and ionization chamber many of the errors usually present in the measurements of the angle are eliminated and the accuracy of the wave-length measurements depends solely on the accuracy with which the circular scale can be read.

This spectrometer is especially suited for absorption measurements for beside determining the critical absorption wave-lengths the magnitude of the absorption may also be estimated. In addition it furnishes a very accurate method of determining Planck's action constant, *h*,

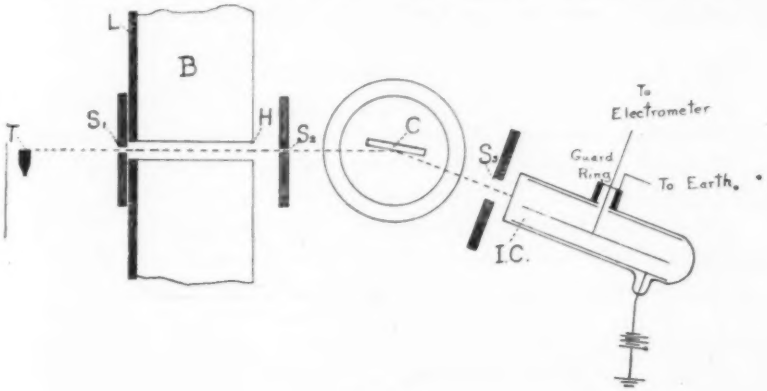

FIG 9

as will be shown later. In making all determinations of wave-lengths the angle through which the crystal table is turned is the angle measured and in this way the two-fold reflection angle is secured. Thus any inaccuracies of measurement are halved in the final determination.

The arrangement of the apparatus is shown in Fig. 9. The x-ray tube, denoted in the figure by *T*, is of the Coolidge hot cathode type and is excited by the current from a 500 cycle A. C. transformer which has been rectified by kenotrons, or by the current from a battery of accumulators.

The tube is separated from the spectrometer by an eighteen-inch brick wall denoted in the figure by *B* and also by lead sheets 1 cm thick

shown at L. A brass tube H serves to allow the rays to pass through the wall. The spectrometer is a modification of an ordinary spectrometer for visible light. Instead of the collimator we have the brass tube H which is closed at either end by a lead disk 1 cm thick which contains an adjustable slit. These slits are denoted by S_1 and S_2 in the diagram and are of the order of a fraction of a mm in width. S_2 takes the place of the objective of the collimator in the ordinary spectrometer. The objective of the telescope is replaced by the slit, S_3, and the telescope itself is replaced by the ionization chamber, $I.C.$ The crystal, C, is carried on a circular table resting on three set screws so that it may be adjusted about a horizontal axis. A micrometer screw fastened to the table serves to move the crystal in a horizontal plane.

Both crystal table and ionization chamber turn about a common vertical axis and their positions are read by two circular scales each carrying two verniers, one at either end of a diameter.

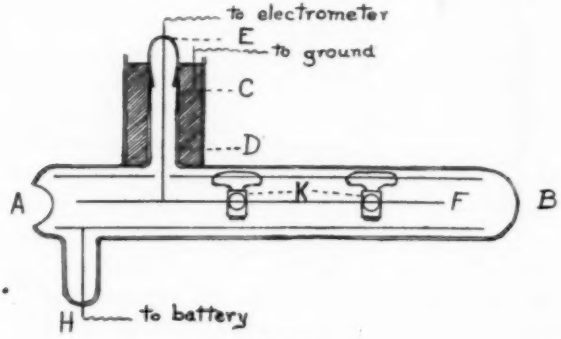

FIG. 10

The ionization chamber is filled with a gas which is easily ionized and since some of the gases, which are desirable, attack grease and cement a specially designed chamber is used in which the gas touches only glass and metal. The details of this chamber are shown in Fig. 10. AB is a glass tube and at A a very thin glass window is blown into it. This window is turned toward the crystal and being very thin absorbs very little of the radiation falling upon it. A guard ring is necessary and to provide this and at the same time secure an air tight joint, a steel tube, C, is ground into a side tube, D, and another glass tube, E, ground into C. A platinum wire passes through E and is fastened to a rod, F, which runs the length of AB. This rod is placed off the center

of AB in order that the beam of x-rays may not strike it. This precaution is necessary in order to avoid the variations in the ionization current which would be produced by the secondary rays coming from the rod, if it were struck by the direct beam. A mercury jacket surrounds the steel tube C and this is earthed. The wire F is one electrode and is connected to the electrometer. The other electrode consists of a thin sheet of steel lying against the inside of AB and connected through H to a battery. The chamber is air tight and may be exhausted by means of a vacuum pump. The stop-cocks, K, are very convenient if the gas used does not attack the stop-cock grease. They are attached to small side-tubes from 5 to 10 cm long, one of which is used to exhaust the chamber, the other for admitting the gas to be employed. If the gas does attack the grease the side-tubes are sealed off after the chamber is filled. In order to minimize electrostatic difficulties the ionization chamber is enclosed entirely in lead with the exception of the thin window at A.

The electrometer used for measuring the relative ionization is ordinarily of the Curie type with a sensitiveness of about 4 meters per volt for a scale distant three meters. It is placed on a stand immediately above the spectrometer and the connecting wire between the ionization chamber and electrometer is imbedded in paraffin and shielded by brass tubes which are earthed. The electrometer itself is shielded by lead plates 1 cm thick. These precautions are necessary in order that the natural leak of the instrument and the ionization due to stray radiation may be reduced to a minimum.

In setting up this spectrometer the x-ray tube is so adjusted that a vertical plane passing through the centers of the slits S_1 and S_2 contains the face of the target. The tube is then shifted slightly until the focal spot on the target face, as seen through the slits S_1 and S_2, appears as a very narrow vertical line. This insures a small angular divergence of the incident beam of rays and at the same time gives a sufficiently large amount of energy in the beam. The slits are adjusted to be as nearly vertical as possible and after the adjustment of the target face S_2 is made very narrow (of the order of a fraction of a millimeter) while S_1 may be removed entirely.

The crystal is placed on the crystal table and adjusted until its front face lies nearly on the axis of rotation of the instrument. The x-ray beam will not be reflected by a single plane in the crystal but by a number of planes lying inside the crystal surface. The effective reflecting plane is defined to be the plane in the crystal such that, if all the x-rays

penetrated to it and were there reflected to the ionization chamber, the effect in the ionization chamber would be the same as the sum of the effects due to the various reflections actually taking place at the successive planes of atoms in the crystal. The effective reflecting plane is the one which should lie on the axis of rotation of the spectrometer though small displacements have no effect on the result if the slits and source are properly arranged. In order to make the adjustment of the reflecting plane a known wave-length is measured on both sides of the zero position of the ionization chamber. This zero position is determined as the point at which the direct beam of rays is received in the chamber and need not be known accurately. If the effective reflecting plane lies on the axis of rotation the angle through which the ionization chamber is turned between the two observations of the known wave-length should be just 4ϕ where 2ϕ represents twice the angle of reflection of the line and is determined from the angle through which the crystal table is turned. If the ionization chamber must be turned through an angle greater than 4ϕ between the two positions of the reflected line it is known that the effective reflecting plane is in front of the axis of rotation, that is, nearer the x-ray tube. If the angle is less than 4ϕ the effective reflecting plane is behind the axis of rotation. It is not necessary to make this adjustment with great precision provided that the ionization chamber slit has sufficient width to allow the entire reflected beam to enter the chamber.

It is necessary that the relative positions of the crystal and ionization chamber be determined so that the ray reflected from the crystal planes to be used shall enter the ionization chamber. To make this adjustment the crystal table is turned about the axis until the reflecting crystal planes make an angle with the direct beam approximately equal to the angle required for the reflection of some wave-length in the spectrum. The ionization chamber is now placed near the point at which the reflected beam should appear and moved by short angular steps through the entire beam. The ionization currents secured, when plotted against ionization chamber angles, gives a curve which starts from near zero, rises to a maximum and again descends nearly to zero. The angle at which the maximum occurs is the correct position of the ionization chamber. By always moving it through twice the angle through which the crystal is turned the relative positions of the two remain correct.

In making a series of wave-length measurements readings corresponding to the same spectral line are taken on both sides of the zero

position. If a reflecting crystal is used the crystal is turned through an angle equal either to $180°+2\phi$ or $180°-2\phi$ in passing from one side of the zero to the other. In case the x-ray beam is transmitted through the crystal, which is frequently true for thin crystals, the crystal is turned through an angle equal to 2ϕ. In any case the magnitude of the angle is easily determined. The angle through which the ionization chamber is turned does not enter into the calculation.

There are four kinds of wave-lengths associated with each series of characteristic x-rays:

(a) Critical ionization wave-lengths. A critical ionization wave-length characteristic of a chemical element is defined as a wave-length such that, if the element is present in a gas through which x-rays pass, x-rays of shorter wave-length than the critical value ionize the gas more strongly than do x-rays of longer wave-length than the critical value.

(b) Critical absorption wave-lengths. A critical absorption wave-length characteristic of a chemical element is a wave-length such that if the element lies in the path of the x-rays it absorbs the rays of wave-length shorter than the critical value more strongly than rays of wave-length longer than the critical value.

(c) Emission wave-lengths. Each series of x-rays contains a number of emission lines of different wave-lengths.

(d) Quantum wave-lengths. A quantum wave-length is a wave-length such that, if it is substituted in the quantum equation, $Ve=h\nu$, it gives the energy of the electron necessary to produce the series of x-rays with which it is associated.

It has been shown that within the limits of experimental error a critical ionization wave-length characteristic of a given element equals the critical absorption wave-length characteristic of the same element and associated with the same series and that the quantum wave-length coincides with the corresponding critical absorption wave-length within the limits of error of the measurements.[10]

In determining an emission wave-length the crystal table is turned about the axis until the approximate position is reached for the reflection of the given line. The ionization chamber is moved into position to receive the reflected ray and both crystal and chamber rotated about the axis, the crystal in steps of $15''$ and the chamber in steps of $30''$. The ionization current is measured in each position and its

[10] W. Duane, Data Relating to X-ray Spectra. Bul. Nat'l Research Council, 1920.

values are plotted as ordinates against crystal table angles as abscissae. A curve similar to the ones shown in Figs. 11a[11] and 11b[12] results. There will be a sharp peak corresponding to each emission line of the

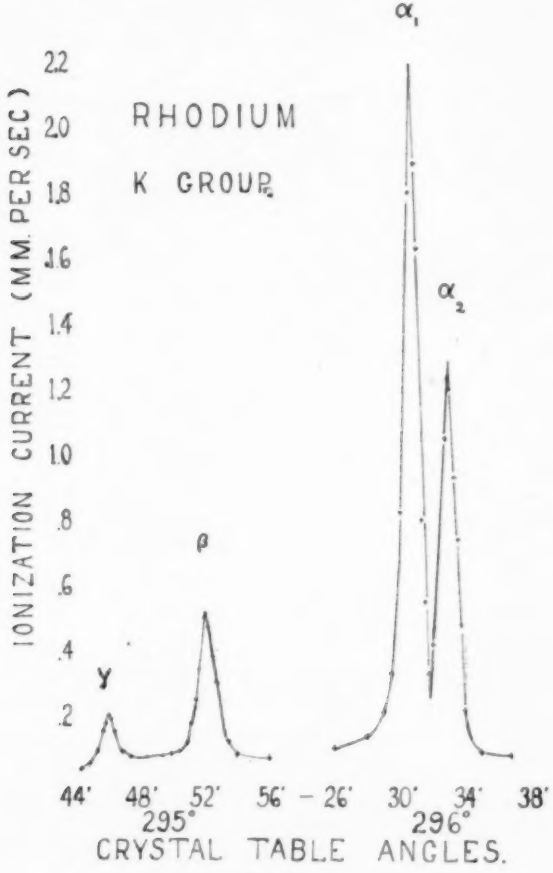

FIG. 11a

spectrum. The position of this peak can be determined as lying between two successive readings and the reading to which it is nearer can also be determined. This gives an error in the location of the peak of some-

[11] Duane and Hu, P. R. *14*, p. 369; 1919.
[12] Duane and Patterson, P. R. *16*, p. 526; 1920.

thing less than 3.8″. If the crystal table is now turned through $2\phi + 180°$ and the ionization chamber through 4ϕ and the readings taken in this new position another peak is secured for the particular line desired. Its position is determined as before to within less than 3.8″. The angle between the two peaks as measured by the turning of the crystal table is accurate to about 7.5″.

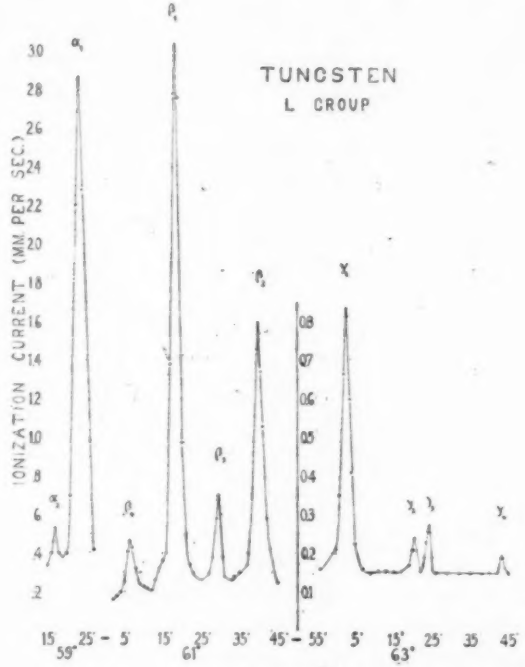

FIG. 11b

In determining a critical absorption wave-length for an element, the element in the form of a thin sheet or powder is placed between slit S_2, and the crystal and the same procedure gone through as with the emission wave-lengths. However, when the ionization currents are plotted against the crystal table angles instead of securing peaks as in the case of emission wave-lengths, sharp drops are secured as shown[13] in Fig. 12. The drops in the curves correspond to the positions of the critical absorption wave-lengths. The center of the straight line drop

[13] Duane, Fricke, Stenstrom, Proc. Nat'l Acad. *6*, p. 607; 1920.

is taken as the correct position of this wave-length in each case and this position may be estimated to within 7.″5 of arc. Since measurements are taken on both sides the zero the errors are additive and 2ϕ is in error about 15″.

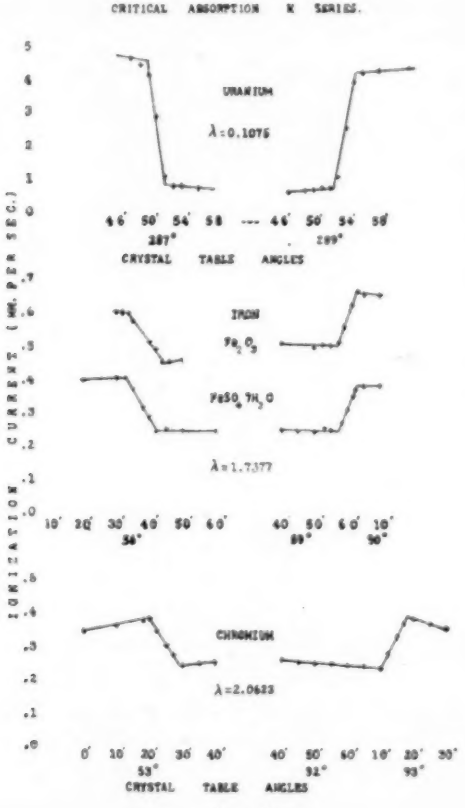

FIG. 12

As stated above some of the errors usually present in the measurement of wave-lengths do not occur in the measurements by this spectrometer. This is due to the arrangement of the slits and source and to the fact that the angle 2ϕ is determined by the positions of the crystal table and not by the positions of the ionization chamber. The arrangement of the slits is shown in Fig. 13. F is the focal spot on the target of the x-ray tube and together with S_2 serves to define the angular width

of the beam. Both are made very narrow (of the order of a fraction of a mm) while slit S_3 is wide enough to admit the entire reflected beam into the ionization chamber. Since the entire beam enters the chamber and the position of the crystal table determines the value of ϕ, it is evident that errors due to the following causes are eliminated; (a) penetration of the rays into the crystal, (b) failure of the effective reflecting plane to coincide exactly with the axis of rotation, (c) the center of the x-ray beam not falling exactly on the axis of rotation, and (d) minor defects in the crystal itself. There remains then only the error in the reading of the angle. The scale can be read correctly to 5″ in the best

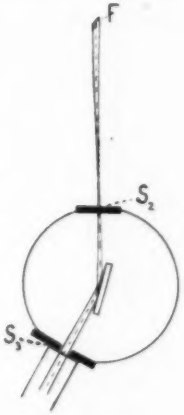

FIG. 13

instrument and since the point of the peak is a result of a number of readings it is safe to consider the peak for each emission wave-length as accurate to 3.8″ as explained above. Since in determining the angle 2ϕ this error is additive, ϕ will be in error by about 3.8″ or for angles in the neighborhood of 2° there will be an error of about .05 per cent.

The absorption wave-lengths are approximately one-half as accurate since 2ϕ can be determined to within but approximately 15″. This gives an error of about 0.1 per cent for values of ϕ near 2°.

In 1915 Duane and Hunt[14] showed that the quantum equation, $Ve = h\nu$ applies to the limit of the continuous x-ray spectrum and used this equation to determine the value of h. V is the maximum potential difference which acts upon the electrons, e is the charge carried by an electron and ν is the frequency of the short wave-length limit of the

[14] Duane and Hunt, P. R. 6, p. 166; 1915.

continuous spectrum. If V and ν are determined accurately, h may be found, e being assumed known.

In the determinations of h and also of the longer wave-lengths the x-ray tube is excited by a high-potential storage battery, the voltage of which may be kept quite constant by means of water resistances. Also the x-ray tube is fitted with a glass side-arm closed at the end by a thin mica window and arranged in such a way that the rays going off from the target almost at grazing incidence pass through the mica window on the way to the crystal. This gives a greater intensity of radiation since the rays pass through less air and through thin mica instead of glass and with greater intensity the slit S_2 may be made narrower thus making smaller the slit correction which must be applied in the h determinations as will be explained later.

The value of ν is the quantity which is determined immediately by means of the spectrometer. This value of ν is the highest frequency which is produced by the particular voltage, V, impressed on the x-ray tube. There are two methods of procedure. Either the crystal table may be set on a given angle corresponding to a known value of ν and the voltage lowered by steps until the ionization current disappears or the voltage may be kept constant and the crystal table and ionization chamber turned about the axis until the point is reached where the ionization current ceases.

In the second method if ionization currents are plotted against crystal table angles curves[15] similar to A and B, Fig. 14 are secured. These two curves represent one determination on either side of the zero. The horizontal portions of the curves correspond to the natural leak of the instrument and the current due to stray radiation. The inclined portions of the curves represent the radiation reflected from the crystal planes. It is seen that these portions of the curves correspond to angular changes of only $2'$ and $3'$ respectively and since the total angle ϕ is about $290'$ the point at which the ionization current ceases can be determined quite definitely. The error in this determination is estimated as about $7''$. While the inclined portion of one of the curves is a straight line and so far as can be seen from the curves continues a straight line until it cuts the horizontal axis, yet a slit correction must be applied due to the fact that the source and slit S_2 are not mathematical lines. The smaller the magnitude of these two widths the smaller the slit correction.

[15] Duane, Palmer, Yeh, J. O. S. A. 5, p. 213; 1921.

The correction to be added to the angle 2ϕ is evidently the greatest angle between any of the rays in the x-ray beam and is calculated from the breadth of the source and slit S_2 since in this set-up slit S_1 is removed. The total angle to be added to 2ϕ is, then, the angular breadth of the slit as seen from the source plus the angular breadth of the source as seen from the slit. The width of the source can be estimated from the marks on the target and in one case[16] this width was 8 mm. Since the slit for this case was distant 760 mm from the target and the rays through the slit made an angle with the target face of about 24′ the angular breadth of the source was 15″.

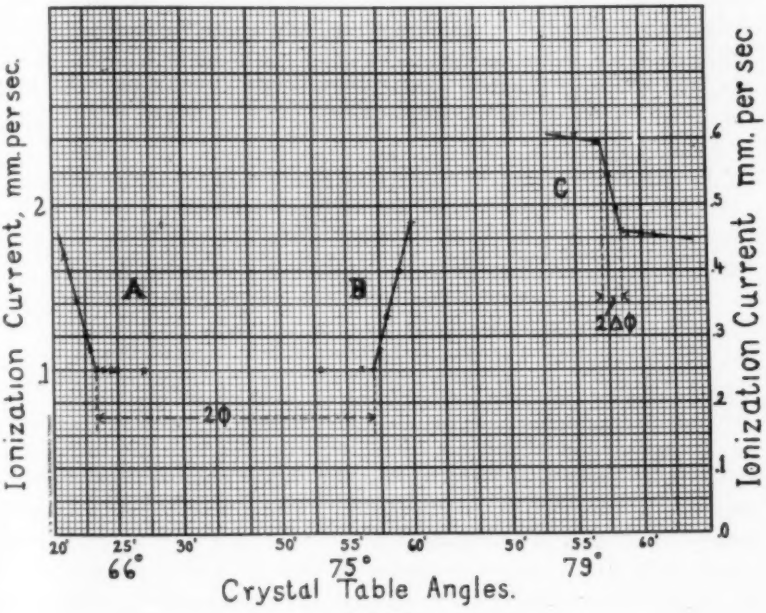

FIG. 14

The actual breadth of the slit together with the distance from source to slit determine the angular slit-breadth. To determine the width of the slit the crystal table is removed and the slits S_2 and S_3 put in line. S_2 is then closed by means of a micrometer screw it carries and the ionization current read for different settings of the micrometer. If ionization currents are plotted as ordinates against micrometer readings

[16] Duane, Palmer, Yeh. J. O. S. A. 5, p. 213; 1921.

as abscissae a straight line results and the point at which this line crosses the axis of abscissae determines the original effective width of the slit.

In the case mentioned above the slit was originally 0.313 mm in width and since source-slit distance was 760 mm the angular slit-width as seen from the target was $1'\,25''$. Then the total correction to 2ϕ was $1'\,40''$. In this connection another method of determining this correction was desired as a check and several emission lines were measured with the thought that the width of the peak would serve as a measurement of the angular divergence of the beam of rays. In every case the width of the peak gave a slightly larger correction than the one calculated. However, when ethyl-bromide was used in the ionization chamber and the rise in the ionization current corresponding to the critical ionization wave-length of bromine was examined it was found that the width of this rise was almost exactly the width of the slit correction as calculated geometrically (Fig. 14*C*). In this case it was found to be $1'42''$.

The difference of potential, V, is determined by a simple potentiometer arrangement in which the voltage across the x-ray tube is compared directly with the electromotive force of a standard cell. The main circuit of the potentiometer consists of a large number of coils of manganin wire having a total resistance of over six million ohms. In the measurements the ratio of two resistances in the main potentiometer circuit gives the ratio of the voltage drop to the electromotive force of the standard cell. The ratio of two resistances can be measured very accurately so that the accuracy of measurement of V is regarded as the same as the accuracy of measurement of the standard cells. All the standard cells used were certified by the Bureau of Standards as accurate to one part in ten thousand. The value of h found as the mean of a number of determinations is 6.556×10^{-27}. If no account is taken of the errors in e, the electronic charge, c, the velocity of light and d, the grating constant of calcite, the error of precision in h is estimated at $\pm.003$. If errors in e, c and d are taken into account the estimated error is $\pm.009$. This value of h is one of the most accurate that has ever been determined and as stated above its determination stands as one of the distinct advantages of the ionization spectrometer. For absorption measurements also this spectrometer seems to be better suited than the photographic types but for long wave-lengths the latter must be used since a vacuum spectrograph is essential.

Several tables of wave-lengths are given in the following pages and all the values have been written to but five significant figures though in

some cases the original values are given to one or two more. In the *L* emission series of the heavy elements there is some disagreement in the results of different authors and so several values are given. Table 10 is taken from a paper by Bohr and Coster [17] and represents the values of the energy levels for the atoms of the different elements. The results given are the wave numbers divided by the Rydberg constant. Values marked in this table with an asterisk are interpolated values or are calculated from interpolated values.

In conclusion I wish to express my appreciation of the many helpful criticisms and suggestions given me by Prof. William Duane in whose laboratory many of the results given in this paper have been secured.

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[17] Bohr, Coster ZS. fur Physik, *12*, p. 342; 1923.

TABLE 1. *Critical Absorption Wave-lengths, K Series* $\lambda \times 10^8$ cm.

Atomic Number	Element	Photographic Method. For Rock Salt $d = 2.814 \times 10^{-8}$ cm.			Ionization Method. For Calcite $d = 3.028 \times 10^{-8}$ cm. Duane, Blake, Hu, Fricke, Stenström and Shimizu.
		de Broglie	Wagner	Fricke	
12	Magnesium			9.5112	
13	Aluminum			7.9470	
15	Phosphorus			5.7580	
16	Sulphur			5.0123	
17	Chlorine			4.3844	
18	Argon			3.8657	
19	Potassium			3.4345	
20	Calcium			3.0633	
21	Scandium			2.7517	
22	Titanium			2.4937	
23	Vanadium			2.2653	
24	Chromium			2.0675	
25	Manganese				1.8892
26	Iron		1.740		1.7396
27	Cobalt				1.6018
28	Nickel		1.485		1.4890
29	Copper	1.388	1.375		1.3785
30	Zinc				1.2963
31	Gallium				1.1902
32	Germanium				1.1146
33	Arsenic				1.0435
34	Selenium	1.003			0.9790
35	Bromine	0.916	0.917		0.9179
36	Krypton	0.8648			
37	Rubidium	0.812			0.8143
38	Strontium	0.767			0.7696
39	Yttrium				0.7255
40	Zirconium	0.684			0.6872

Atomic Number	Element	de Broglie	Wagner	Siegbahn-Jönsson	Duane, Blake, Hu, Fricke, Stenström and Shimizu.
41	Niobium	0.648			0.6503
42	Molybdenum	0.614			0.6184
44	Ruthenium				0.5584
45	Rhodium				0.5330
46	Palladium	0.505	0.513		0.5075
47	Silver	0.482	0.484		0.4850
48	Cadmium	0.460	0.462	0.4629	0.4632
49	Indium				0.4434
50	Tin	0.421	0.422	0.4231	0.4242
51	Antimony	0.401	0.405		0.4065
52	Tellurium	0.385	0.383	0.3877	0.3896
53	Iodine	0.369	0.369	0.3715	0.3737
54	Xenon	0.3588			
55	Caesium	0.340		0.3436	0.3444
56	Barium	0.327	0.331	0.3306	0.3307
57	Lanthanum	0.313		0.3186	0.3188
58	Cerium	0.300	0.298	0.3064	0.3068
59	Praseodymium			0.2946	
60	Neodymium		0.282	0.2835	0.2861
62	Samarium			0.2636	
63	Europium			0.2543	
64	Gadolinium			0.2456	
65	Terbium				0.2398
66	Dysprosium			0.2294	0.2308
67	Holmium			0.2214	
			Martin		
68	Erbium		0.2159		
69	Thulium	0.208			
70	Neo-Ytterbium	0.2015			
71	Lutecium	0.195			
72	Hafnium	0.1905			
74	Tungsten				0.1781
76	Osmium				0.1683
78	Platinum	0.152		0.1578	0.1581
79	Gold	0.149		0.1524	0.1534
80	Mercury	0.146		0.1479	0.1491
81	Thallium	0.142		0.1427	0.1448
82	Lead	0.138		0.1385	0.1410
83	Bismuth	0.133		0.1346	0.1372
90	Thorium			0.1127	0.1131
92	Uranium			0.1048	0.1075

References: de Broglie, Journ. de Physique, p. 161; May-June, 1916, and C. R. *170*, p. 725; 1920; Wagner, Ann. der Phys. *46*, p. 868; 1915; Blake and Duane, P. R. *10*, pp. 98 and 698; 1917. Duane and Hu, P. R. *11*, p. 488; 1918; and *14*, p. 516; 1919; Duane and Shimizu, P. R. *13*, p. 159 and *14*, p. 523; 1919. Siegbahn and Jönsson, Physik. Zeit. *20*, p. 251; 1919. Fricke, P. R. *16*, p. 202; 1920. Duane, Fricke and Stenstrom, Nat. Acad. Proc. Sept. 1920. de Broglie and Lepape, C. R. *176*, p. 1611; 1923. Martin, Roy. Soc. Victoria Proc. *35*, 2. p. 164. de Broglie and Caprera C. R. *176*, p. 433; 1923.

TABLE 2. *Critical Absorption Wave-lengths in the L Series* $\lambda \times 10^8$ cm.

Photographic Method; (H) Hertz, (L) Lindsay, (C) Cork, (B) de Broglie (D) Dauvillier and Lepape. (Co) Coster. Ionization Method: (DP) Duane and Patterson.

Atomic Number	Element	L_1	L_2	L_3	Author
51	Antimony	2.9945	2.8310	2.6327	L
52	Tellurium	2.8410	2.6837	2.5026	L
53	Iodine	2.7124	2.5483	2.3819	L
54	Xenon	2.5875	2.4253	2.2724	D
55	Caesium	2.4678	2.3073	2.1605	L
56	Barium	2.3577	2.1995	2.0602	L
57	Lanthanum	2.250	2.098	1.971	H
58	Cerium	2.158	2.007	1.887	H
59	Praseodymium	2.071	1.922	1.808	H
60	Neodymium	1.992	1.842	1.736	H
62	Samarium	1.8410	1.6905	1.5996	D
63	Europium	1.7647	1.5896	1.5337	D
64	Gadolinium	1.699	1.550	1.470	C
66	Dysprosium	1.576	1.435	1.362	C
68	Erbium	1.478	1.336	1.265	C
70	Ytterbium	1.386	1.242	1.171	C
72	Hafnium	1.2930	1.1515	1.097	Co
73	Tantalum	1.253	1.1118	1.058	C
74	Tungsten	1.2136	1.0726	1.024	DP
76	Osmium	1.138	0.9985	0.9515	C
77	Iridium	1.1036	0.965	0.9195	C
78	Platinum	1.0704	0.9321	0.8885	DP
79	Gold	1.0383	0.8993	0.8606	DP
80	Mercury	1.0067	0.8700	0.8335	DP
81	Thorium	0.9776	0.8415	0.8055	DP
82	Lead	0.9497	0.8133	0.7803	DP
83	Bismuth	0.9216	0.7872	0.7532	DP
88	Radium	0.802	0.668		B
90	Thorium	0.7596	0.6286	0.6044	DP
92	Uranium	0.7214	0.5918	0.5685	DP

References: Hertz, ZS. f. Phys. *3*, p. 19; 1920; de Broglie, Journ. de Phys. May-June 1916 and p. 31; Jan. 1919; Duane and Patterson, Nat. Acad. Proc.; Sept. 1920. Cork, P. R. *21*, p. 326; 1923. Lindsay, C. R. *175*, p. 150; 1922. A. Lepape and A. Dauvillier, C. R. *177*, p. 34; 1923.

TABLE 3. *Critical Absorption Wave-lengths in the M Series.* $\lambda \times 10^8$ *cm.*
Photographic Method by Stenström (S) and Coster (C).

Atomic Number	Element	M_1	M_2	M_3	M_4	M_5	Author
83	Bismuth	4.762	4.569	3.894			C
90	Thorium	3.721	3.552	3.058			S
90	Thorium				2.571	2.388	C
92	Uranium	3.491	3.326	2.873			S
92	Uranium				2.385	2.228	C

References: Stenström, Doctor's Dissertation, Lund, 1919.
Coster, P. R. *19*, p. 20; 1922.
Also Ross, P. R. *22*, p. 221; 1923.

TABLE 4. *Emission Wave-lengths, K Series* $\lambda \times 10^8$ *cm.*

At. No.	Element	$\alpha_1\ \alpha_2$		α_1'	α_3	α_4	α_5	α_6	β'	β_1	β_2
11	Sodium	11.884		11.835	11.802	11.781				11.591	
12	Magnesium	9.8678		9.8265	9.7994	9.7862	9.7302	9.7118	9.647	9.5345	
13	Aluminium	8.3194		8.2856	8.2646	8.2530	8.2058	8.1892	8.025	7.9405	
14	Silicon	7.1092		7.083	7.0638	7.0537	7.014	7.003	6.7933	6.7393	
15	Phosphorus	6.1417			6.1022	6.0950			5.8204	5.7862	5.8202
		α_2	α_1	α_1'	$(\alpha_3\alpha_4)$		β'	β_1	β''	γ'	β_3
16	Sulphur	5.3638	5.3613	5.3406	$\alpha_2$5.3294 $\alpha_4$5.3233		5.047	5.0213		5.0130	5.0447
17	Chlorine	4.7219	4.7182		4.686			4.3946	4.3908		4.4060
19	Potassium	3.7373	3.7337	3.7187	3.710			3.4468	3.4429	3.4341	
20	Calcium	3.3551	3.3517	3.3399	3.331		3.0911	3.0834	3.0796	3.0691	
21	Scandium	3.0286	3.0250		3.006		2.7992	2.7739		2.7582	
22	Titanium	2.7465	2.7432		2.7269		2.5151	2.5090		2.4937	
											γ_1
23	Vanadium	2.5021	2.4984		2.4846		2.2853	2.2797			2.2646
24	Chromium	2.2896	2.2848		2.2733		2.0857	2.0803			2.0670
25	Manganese		2.0973		2.0879		1.9105	1.9059			1.8932
26	Iron	1.9366	1.9323		1.9233		1.7560	1.7530			1.7406
27	Cobalt	1.7901	1.7853		1.7774		1.6197	1.6171			1.6054
28	Nickel	1.6586	1.6546		1.6476		1.4990	1.4970			1.4854
29	Copper	1.5412	1.5373		1.5308			1.3893			1.3780
30	Zinc	1.4360	1.4321		1.4288			1.2927			1.2811
		α_2		α_1	β		γ				
31	Gallium	1.3416		1.3379	1.2059						
32	Germanium	1.261		1.257	1.131		1.121				
33	Arsenic	1.1775		1.1735	1.052		1.038				
34	Selenium	1.109		1.104	0.993						
35	Bromine	1.040		1.035	0.929		0.914				
37	Rubidium	0.926		0.922	0.825		0.813				
38	Strontium	0.876		0.871	0.779		0.767				
39	Yttrium	0.840		0.835	0.746		0.733				

		α_2	α_1	β	γ	
40	Zirconium	.793	.788	.705		
41	Niobium	.754	.749	.669	.657	
42	Molybdenum	.71212	.70783	.63110	.6197	D
44	Ruthenium	.6454	.6410	.5705	.5593	A and D
45	Rhodium	.61636	.61210	.54527	.5342	D
46	Palladium	.58850	.58421	.51962	.51021	
47	Silver	.56251	.55816	.49597	.491	
48	Cadmium	.53837	.53395	.47414	.4647	
49	Indium	.515	.510	.453	.440	
50	Tin	.49385	.48947	.432		
51	Antimony	.472	.468	.416	.408	
52	Tellurium		.456	.404		
53	Iodine	.4417	.4368	.3878	.3818	D and C
55	Caesium	.402	.398	.352		
56	Barium	.393	.388	.343		
57	Lanthanum	.376	.372	.329		
58	Cerium	.360	.355	.314		
59	Praseodymium	.347	.342	.301		
60	Neodymium	.335	.330	.292		
74	Tungsten	.21341	.20860	.18420	.17901	D
77	Iridium	.1958		.1684		S and L
78	Platinum	.19010	.18528	.1634	.1582	

References: (D) W. Duane and co-workers, Bull. Nat'l Research Council, Data Relating to X-ray Spectra, Nov., 1920. (D. and C.) W. Duane and G. L. Clark, Proc. Nat'l Acad. Dec., 1923 and Jan., 1924. (A. and D.) Auger and Dauvillier, C. R. *176*, p. 1297; 1923. (S. and L.) Seemann and Lilienfeld, Phys. Zeit., *19*, p. 269; 1918. All values not designated are by Siegbahn and his co-workers at Lund. M. Siegbahn, Jahr. d Radioak. u. Elektronik, *18*, p. 240; 1921, and Siegbahn and Dolejšek, ZS. f. Phys. *10*, p. 159; 1922. Siegbahn, Spektroskopie der Röntgenstrahlen p. 101.

TABLE 5. *Emission Wave-lengths in the L Series* $\lambda \times 10^8$ *cm.*

	α_3	α_1	α'_1	α_2	α_4	β_1	β'_1	β_{10}
29 Cu		13.309						
30 Zn		12.223				11.951		
32 Ge		10.414						
33 As		9.6503		9.6173		9.3940		
34 Se		8.9706		8.9386		8.7172		
35 Br		8.3566		8.3262		8.1076		
37 Rb		7.3027		7.2710	7.2488	7.0604	7.041	7.0224
38 Sr		6.8478		6.8183	6.7973	6.6092	6.590	6.5734
39 Y		6.4349		6.4065		6.1984		
40 Zr		6.0559		6.0291	6.0132	5.8228	5.807	5.7933
41 Nb	5.717	5.7113		5.6849	5.6711	5.4796	5.465	5.4497
42 Mo	5.400	5.3943		5.3703	5.3564	5.1658	5.150	5.1384
44 Ru	4.8437	4.8357		4.818		4.6110		
45 Rh	4.5956	4.5878	4.5649	4.5713	4.5588	4.3640	4.351	4.3425
46 Pd	4.3666	4.3585	4.3362	4.3427	4.3309	4.1373	4.125	4.1171
47 Ag	4.1538	4.1456	4.1254	4.1310	4.1194	3.9266	3.914	3.9069
48 Cd	3.9564	3.9478	3.9288	3.933		3.7301	3.719	3.7116
49 In	3.7724	3.7637	3.7447	3.7499		3.5478	3.539	3.5304
50 Sn	3.6011	3.5922	3.5740			3.3779	3.367	3.3607
51 Sb	3.4408	3.4318	3.4138			3.2184	3.210	3.2026
52 Te	3.2910					3.0700		
53 I	3.1509					2.9309		
55 Cs	2.8956					2.6778		
56 Ba	2.7790					2.5622		
57 La	2.6689					2.4533		
58 Ce	2.5651					2.3510		
59 Pr	2.4676					2.2539		
60 Nd	2.3756					2.1622		

TABLE 5. (*Continued.*) *Emission Wave-lengths in the L Series* $\lambda \times 10^8$ *cm.*

	β_4	β_3	β_6	β_2	β_5	β_{11}	β_{12}	β_7
29 Cu								
30 Zn								
32 Ge								
33 As								
34 Se								
35 Br								
37 Rb	6.8028	6.7699	6.9675					
38 Sr	6.3855	6.3499	6.5053					
39 Y	6.0019	5.9678						
40 Zr	5.6527	5.6182	5.6935	5.5734				
41 Nb	5.3314	5.2959	5.3461	5.2253		5.161	5.161	
42 Mo	5.0358	5.0002		4.9092		4.8597	4.8417	
44 Ru	4.5126	4.4764	4.4764	4.3619				
45 Rh	4.2778	4.2413	4.2301	4.1221		4.0848	4.0725	
46 Pd	4.0623	4.0257	4.0070	3.9007		3.8676	3.8567	3.676
47 Ag	3.8611	3.8245	3.7994	3.6938		3.6633	3.6537	3.4795
48 Cd	3.6743	3.6364	3.6073	3.5064		3.4775	3.4684	3.302
49 In	3.499		3.4280	3.3312		3.3040	3.2959	3.125
50 Sn	3.3363	3.2989	3.2622	3.1679		3.1426	3.1347	2.9685
51 Sb	3.1843	3.1451	3.1078	3.0166		2.9934	2.9858	
52 Te	3.0400	3.0013	2.9644	2.8761				
53 I	2.906	2.867		2.7461				
55 Cs	2.6605	2.6229	2.5875	2.5064				
56 Ba	2.5498	2.5110	2.4772	2.3993				2.218
57 La	2.4443	2.4053		2.298				
58 Ce	2.3448	2.3059		2.2038	2.1960			
59 Pr	2.254	2.2124		2.1147				
60 Nd	2.162	2.1223		2.031				

TABLE 5. (*Continued.*) *Emission Wave-Lengths in the L Series* $\lambda \times 10^8$ *cm.*

	γ_0	γ_1	γ_2	γ_3	γ_4	l	η
29 Cu							
30 Zn							
32 Ge							
33 As							
34 Se							
35 Br							
37 Rb	6.7386		6.0282				8.0290
38 Sr	6.2788		5.6294			7.821	7.505
39 Y							
40 Zr	5.4810	5.3730	4.9412			6.898	6.5931
41 Nb		5.0241	4.639			6.509	6.195
42 Mo	4.8190	4.7111	4.3613				5.835
44 Ru	4.2766	4.1728	3.8879				4.9112
45 Rh	4.0352	3.9357	3.6770			5.2070	4.6502
46 Pd	3.8116	3.7164	3.4809			4.9396	4.4101
47 Ag	3.6073	3.5149	3.2997			4.6976	4.1875
48 Cd	3.4181	3.3280	3.1316			4.4713	3.9761
49 In	3.2418	3.1553	2.9736		2.9191	4.2593	3.7818
50 Sn	3.0774	2.9949	2.8273		2.7713	4.0633	3.5996
51 Sb	2.9256	2.8451	2.6889		2.6336	3.8803	
52 Te	2.7831	2.7065	2.5649		2.5057	3.7101	
53 I		2.5775					
55 Cs		2.3425	2.2322	2.2270	2.1691		2.9833
56 Ba	2.3023	2.2366	2.1340	2.1296	2.0715	3.1287	2.8571
57 La		2.1372				3.000	
58 Ce		2.0443					
59 Pr		1.9568				2.7881	
60 Nd		1.8738				2.6703	

At. No.	α_2	α_1	β_4	β_1	β_3	β_2	β_5	γ_1	γ_2	γ_3	γ_4
62 Sm	2.2057	2.1950		1.9936	1.958	1.877		1.7231	1.657		
63 Eu	2.1273	2.1163	1.921	1.9163	1.886	1.807		1.659	1.598		
64 Gd	2.0526	2.0419	1.848	1.8425	1.809	1.741		1.5886			
65 Tb	1.9823	1.9715	1.781	1.7727	1.7426	1.6793	1.656	1.529	1.474	1.468	1.434
66 Dy	1.9156	1.9046	1.718	1.7066	1.680	1.6198		1.467	1.419	1.415	
67 Ho	1.8521	1.8410	1.653	1.6435	1.6168	1.5637		1.412	1.366	1.361	
68 Er	1.7914	1.7804	1.596	1.5834	1.556	1.5112		1.363	1.320	1.313	
70 Yb	1.6789	1.6678	1.488	1.472	1.448	1.412		1.265	1.225	1.221	
71 Lu	1.6264	1.6155	1.434	1.417	1.395	1.366		1.220	1.184	1.180	

References: Results are by Siegbahn and his co-workers at Lund.
Jahr. d. Radioak, u. Elek. *18*, p. 280; 1921.
A. Žáček, ZS. f. Phys. *15*, p. 31; 1923.
D. Coster P.M., *43*, p. 1070, 1922; ZS. f. Physik *6*, p. 185; 1921.

TABLE 6. *Emission Wave-lengths in the L Series of Tungsten.* $\lambda \times 10^8$ *cm.*

	Photographic Methods For Rock-salt $d = 2.814 \times 10^8$ cm.				Ionization Method For Calcite $d = 3.028 \times 10^8$ cm.		
line	Gorton	Dershem	Overn	Siegbahn	Coster	A. H. Compton	Duane and Patterson
l				1.6751			1.6756
α_2	1.476	1.4828	1.4839	1.4845		1.4840	1.4839
α_1	1.466	1.4722	1.4731	1.4735		1.4728	1.4731
η		1.4163		1.4177			1.4176
						1.3360	
β_4	1.292	1.2977	1.2984	1.2987		1.2982	1.2985
β_6	1.283	1.2868	1.2872	1.2871			
β_1	1.275	1.2784	1.2793	1.2792		1.2792	1.2789
β_3	1.256	1.2586	1.2598	1.2600		1.2598	1.2601
β_2	1.237	1.2416	1.2434	1.2419		1.2416	1.2419
β_8			1.2355		1.2354		
β_7		1.2202	1.2212	1.2205	1.2208	1.2183	
β_5			1.2132	1.2118	1.2125		
β_{10}		1.2098	1.2097		1.2094		
β_9		1.1773	1.2021	1.2031	1.2021		1.2040
γ_5		1.1292	1.1302	1.1284	1.1292		
γ_1	1.094	1.0953	1.0967	1.0955		1.0961	1.0961
			1.0794				
γ_6		1.0705	1.0724		1.0720		
γ_2		1.0648	1.0659	1.0658		1.0650	1.0655
γ_3	1.057	1.0587	1.0596	1.0597		1.0580	1.0596
		1.0427	1.0446				
γ_4	1.025	1.0253	1.0263	1.0265		1.0247	1.0261

References: W. S. Gorton, P. R., *7*, pp. 203 and 334; 1916. A. H. Compton, P.R., *7*, p. 646; 1916. E. Dershem, P.R., *11*, p. 461; 1918. O. B. Overn, P.R. *14*, p. 137; 1919. M. Siegbahn, P.M. *38*, p. 639; 1919. Jahr. d. Radioak, u. Elek. *18*, p. 284; 1921. Duane and Patterson, P.R., *15*, p. 328; 1920.

TABLE 7. *Emission Wave-lengths in the L Series of Some of the Heavy Elements by Coster* $\lambda \times 10^8$ *cm.*

At. No.	l	α_2	α_1	η	β_4	β_6	β_1	β_3	β_2
69 Tm		1.7339	1.7228	1.6923	1.5412	1.5115	1.5268	1.5023	1.4602
72 Hf	1.7774	1.5770	1.5661	1.5197	1.3893	1.3711*	1.3711*	1.3497	1.3235
73 Ta	1.7242	1.5294	1.5183	1.4655	1.3422	1.3267	1.3235	1.3033	1.2810
76 Os		1.3982	1.3882		1.2150	1.2048	1.1946	1.1772	1.1684
77 Ir		1.3594	1.3483		1.1764	1.1717	1.1550	1.1379	1.1329
78 Pt	1.4972	1.3212	1.3101	1.2401	1.1398*	1.1398*	1.1172	1.0995*	1.0995*
79 Au	1.4565	1.2849	1.2736	1.1995	1.1044	1.1106	1.0809		1.0678
81 Tl		1.2160	1.2047	1.125	1.0371	1.0480	1.0127	.9978	1.0079
82 Pb	1.3466	1.1835	1.1720	1.0902	1.0047	1.0188	.97990*	.96602	.97990*
83 Bi	1.3130	1.1533	1.1412	1.057	.9754	.9916	.94930	.9357	.95293
90 Th	1.1124	.96524	.95342		.789	.8262	.76259*	.7521	.79108
92 U	1.0648	.92014	.90833	.8029	.7454	.78656	.71807	.7084	.75268

At. No.	β_8	β_7	β_5	γ_3	γ_1	γ_4	γ_2	γ_3	γ_4
69 Tm				1.3523	1.3127		1.2712	1.2653	1.2264
72 Hf		1.3035		1.2121	1.1765		1.1413	1.1356	1.1001
73 Ta	1.2738	1.2600	1.2506	1.1700	1.1347	1.1100	1.1020	1.0962	1.0624
76 Os			1.140		1.0225				
77 Ir	1.1267		1.1030		.98841	.9636*	.9636*	.9566	
78 Pt	1.0928	1.0785	1.0701	.9855	.95545	.9317*	.9317*	.9256	.8950
79 Au	1.0609	1.0465	1.0382	.9542	.92437	.90125*	.90125*	.89568	.8663
81 Tl		.988	.9783	.8942	.86529	.8417	.8447	.8379	.8100
82 Pb	.9735	.9590	.94952	.8639	.83708	.81370*	.8182	.81370*	.7836
83 Bi			.9223	.8378	.81065	.7874*	.7929	.7874*	.761
90 Th			.76259*		.65103	.6301			
92 U		.736	.72413		.61283	.5926	.6044	.5970	

Lines marked * lie so near together that they cannot be distinguished.

Reference: D. Coster, ZS. f. Physik; *4*, p. 178, 1921, *5*, p. 139; 1921. P.M. *46*, p. 956; 1923

TABLE 8. *Wave-lengths of the L Series of Some of the Heavy Elements and Their Separation into the L_1, L_2, and L_3 groups by Dauvillier.*

L_1

At. No.	β_5'	β_5	β_5''	β_7	β_7'	β_2''	β_2'''	β_2	β_2'
92 U		.7242		.7340	.7364	.7459		.7527	.7547
79 Au	1.0360	1.0380	1.0412	1.0453	1.0472	1.0610	1.0623	1.0675	1.0695
78 Pt		1.0700		1.0762	1.0790	1.0941		1.0996	1.1010
77 Ir		1.1033		1.1082	1.1116	1.1273		1.1329	
76 Os		1.1389			1.1479	1.1616		1.1678	1.1696
74 W		1.2146			1.2224	1.2360		1.2424	
73 Ta		1.2518			1.2613	1.2750		1.2803	
71 Lu		1.3998		1.3466		1.3579		1.3668	1.3685
70 Yb						1.4059		1.4124	
64 Gd		1.7093			1.7200	1.7293		1.7420	
63 Eu		1.7734			1.7817	1.7940		1.8072	
62 Sm		1.8437			1.8536	1.8662		1.8788	
60 Nd					2.0066	2.0138		2.0322	
58 Ce		2.1640			2.1770	2.1844		2.2040	
56 Ba		2.3595			2.3717	2.3813		2.3813	
51 Sb		2.9677				3.0117		3.0164	

TABLE 8. (*Continued*) *Wave-lengths of the L Series of Some of the Heavy Elements and Their Separation into the L_1, L_2, and L_3 groups by Dauvillier.*

L_1

At. No.	β_{14}	β_6	α_2	α_1	α_2	s	l	l
92 U		.7859				.9617		
79 Au		1.1087	1.2689	1.2728		1.3481	1.4100	
78 Pt		1.1400	1.3049	1.3100	1.3210		1.4490	
77 Ir		1.1773	1.3433	1.3485	1.3595	1.4295	1.4900	
76 Os		1.2107	1.3832	1.3883	1.3993			
74 W		1.2883				1.5610	1.6216	
73 Ta		1.3280	1.5137	1.5177	1.5289	1.6086	1.6717	1.7235
71 Lu		1.3984	1.6122	1.6176				
70 Yb		1.4624						
64 Gd	1.7487	1.8020						
63 Eu	1.8139	1.8689						
62 Sm	1.8860	1.9430	2.1846	2.1950	2.2054			
60 Nd	2.0395	2.0997						
58 Ce	2.2124	2.2772						
56 Ba								
51 Sb			3.4177					

L_2

At. No.	γ_2'	γ_2	γ_6	γ_{10}	γ_1	γ_1'	γ_5	β_9	β_{13}	β_1	β_{11}	η
92 U		.5923			.6128		.6344			.7181		.8028
79 Au	.8990	.9012	.9075		.9244		.9533			1.0809		
78 Pt		.9316	.9379	.9503	.9554		.9851			1.1172	1.1634	
77 Ir		.9637	.9714	.9822	.9884		1.0197			1.1549	1.2044	1.2836
76 Os		.9991	1.0053		1.0225		1.0541			1.1946		1.3258
74 W		1.0720	1.0785	1.0887	1.0955		1.1288			1.2788	1.3344	
73 Ta		1.1107	1.1176	1.1290	1.1355		1.1700			1.3231	1.3832	1.4672
71 Lu		1.1969	1.2016	1.2119	1.2194		1.2568			1.4205		1.5754
70 Yb		1.2405	1.2483	1.2568	1.2650		1.3038			1.4726		
64 Gd		1.5615	1.5695	1.5789	1.5886	1.5946	1.6375	1.7033	1.8351	1.8425		
63 Eu		1.6241	1.6334	1.6422	1.6535	1.6589	1.7046	1.7647	1.9081	1.9154		
62 Sm		1.6926	1.7019	1.7113	1.7227	1.7287	1.7763	1.8340	1.9876	1.9948		2.2145
60 Nd		1.8246	1.8514	1.8600	1.8740	1.8793	1.8793	1.8793	2.1548	2.1622		
58 Ce		2.0091	2.0196	2.0288	2.0455	2.0511	2.1060	2.1531		2.3523		
56 Ba		2.2016	2.2112	2.2186				2.3511				
15 Sb				2.840	2.8451					3.2216		3.5070

L_3

At. No.	γ_8	γ_4	γ_9	γ_3	γ_3'	γ_7	γ_7'	β_8	β_9	β_5	β_4
92 U	.5689	.5738		.5968		.6038		.6797	.6853	.7086	.7459
79 Au		.8658	.8830	.8961		.9025		1.0182	1.0254	1.0652	1.1041
78 Pt		.8940	.9120	.9258		.9316		1.0524	1.0593	1.0996	1.1400
77 Ir		.9245	.9426	.9568		.9623		1.0874	1.0977	1.1384	1.1773
76 Os		.9572	.9750	.9888		.9957		1.1238		1.1774	1.2159
74 W		1.0262	1.0439	1.0592		1.0656		1.2034	1.2105	1.2598	1.2985
73 Ta				1.0970		1.1024		1.2430	1.2518	1.3030	1.3423
71 Lu		1.1414	1.1605	1.1765		1.1826		1.3330	1.3398	1.3984	1.4372
70 Yb		1.1826	1.2016	1.2194		1.2262				1.4488	1.4886
64 Gd		1.4815		1.5269		1.5336	1.5383	1.7200	1.7293	1.8113	1.8501
63 Eu		1.5410	1.5619	1.5875	1.5919	1.5953	1.5986	1.7860	1.7940	1.8815	
62 Sm		1.6032	1.6261	1.6517	1.6560	1.6604	1.6644	1.8589	1.8662	1.9588	1.9948
60 Nd						1.9634		2.0138		2.1224	2.1622
58 Ce		1.8948	1.9225	1.9515	1.9561			2.1844		2.3061	2.3452
56 Ba								2.3755	2.3813		
51 Sb		2.6332		2.6878		2.6946		2.9855	2.989	3.1463	3.1853

References: de Broglie, Les Rayons X. Dauvillier, Numerous References in C. R., p. 172 1921, to p. 176, 1923.

TABLE 9. *Emission Wave-lengths in the M Series.* $\lambda \times 10^8$ *cm.*

Photographic Method. For Calcite $\log 2d = 0.78233 - 8$
Sugar $\log 2d = 1.32512 - 8$
Gypsum $\log 2d = 1.18056 - 8$

At. No.	M_5P_1	M_5O_3	M_4O_2	M_5N_3	M_4N_4	M_5N_6	M_2O_1	M_3O_5	M_4N_7	γ'	γ
71 Lu											6.780
73 Ta											6.301
74 W										6.066	6.085
76 Os				4.779	4.949				5.802	5.652	5.672
77 Ir				4.548	4.768						5.484
78 Pt				4.407							5.303
79 Au				4.230							5.131
81 Tl				3.932	4.095					4.798	4.806
82 Pb				3.789	3.945				4.646		4.666
83 Bi				3.672	3.816	3.884				4.497	4.513
90 Th		2.437	2.612	2.917	2.999	3.127	3.109	3.276	3.530	3.645	3.657
92 U	2.248	2.299	2.439	2.750	2.815	2.909	2.927	3.107	3.321	γ'' 3.466	3.472
										γ' 3.459	

Emission Wave-lengths in the M Series (Continued)

At. No.	M_1P_1	M_2O_4	β'	β''	β	α'	α''	α_1	α_2	M_3N_7	M_1N_2
66 Dy					9.323						
67 Ho				8.919	8.943			9.150			
68 Er					8.573			8.783			
70 Yb			7.852	7.870	7.891	8.090	8.111	8.125			
71 Lu				7.560	7.582	7.787	7.803	7.820			
73 Ta					7.001			7.238			
74 W				6.726	6.745		6.952	6.973			
76 Os				6.233	6.256		6.459	6.481		6.882	
77 Ir				6.011	6.030		6.223	6.250		6.663	8.012
78 Pt				5.797	5.820		6.026	6.041			
79 Au					5.619		5.812	5.831		6.264	
81 Tl		5.185		5.210	5.233		5.427	5.443		5.879	
82 Pb		4.994		5.042	5.065	5.242	5.250	5.273		5.687	6.727
83 Bi		4.815		4.875	4.894		5.078	5.107	5.117	5.525	6.498
90 Th	3.753	3.792	3.921	3.925	3.931		4.097	4.129	4.138	4.569	5.245
92 U	3.514	3.570	3.684	β''' 3.700	3.709		3.885	3.901	3.913	4.326	4.929
				β'' 3.696							

M_4N_4 means the line produced when an electron passes from the N_4 energy level to the M_4 energy level.

Reference: Hjalmar, Doctor's Dissertation, Lund, 1923. See also Stenström, Doctor's Dissertation, Lund, 1919. Karcher, P. R. *15*, p. 285; 1920. Ross, P. R. *22*, p. 221; 1923.

TABLE 9A. *Measurements in the N Series* $\lambda \times 10^8$ *cm.*

At No.	Element	N_7P_1	N_7O_2	N_6O_2	N_5P_2	N_5O_1
92	Uranium	8.691	9.619	10.385	12.250	12.874
90	Thorium	9.397	10.030	11.046	13.149	13.805
83	Bismuth	13.208				

Reference: Hjalmar, Dissertation, Lund, 1923. See also Dolejšek, ZS. f. Physik, *10*, p. 129; 1922.

TABLE 10. *Energy Level Values According to Bohr and Coster.*

	K	L_I	L_{II}	L_{III}	M_I	M_{II}	M_{III}	M_{IV}	M_V	N_I	N_{II}	N_{III}	N_{IV}	N_V	N_{VI}	N_{VII}	O_I	$O_{II, III}$	$O_{IV, V}$	
92 U	8477.0	1603.5	1543.1	1264.3	408.9	382.1	317.2	274.0	261.0	106.6	95.7	77.1	56.3	53.6	28.4	27.6	26.2	15.4	5.8	92 U
90 Th	8073.5	1509.7	1451.5	1200.6	381.6	354.4	298.0	256.6	244.9	97.8			51.2	48.7	24.8	24.1			5.7	90 Th
83 Bi	6646.7	1207.9	1159.4	990.0	295.9	273.6	234.0	199.4	191.4	71.0	58.7	50.3	35.7	33.7	13.6	13.0		11.0	2.0	83 Bi
82 Pb	6463.0	1169.3	1121.9	960.5	283.8	262.3	226.0	190.5	183.0	66.0	55.4	49.3	32.2	30.5	10.8	10.3	10.3	6.4	0.8	82 Pb
81 Tl	6289.0	1132.4	1084.2	933.2	273.9	253.8	219.2	184.8	176.8	63.7	53.6	44.9	30.6	29.0	10.0	9.6	10.6	7.4	1.7	81 Tl
80 Hg	6115.9	1094.6	1048.6	906.1		[redacted]														80Hg
79 Au	5940.4	1060.2	1014.4	878.5	252.9	235.1	202.8	169.3	163.0	58.0	49.1	42.8	26.4	25.0	6.4		7.8	8.3	0.8	79 Au
78 Pt	5764.0	1026.8	978.7	852.0	243.4	227.3	198.0	162.3	156.4	52.5	48.7	42.3	24.5	23.2	5.2		7.1	8.6	0.4	78 Pt
74 W	5113.8	890.6	850.1	751.6	207.6	188.9	167.4	137.8	133.2	43.6	35.6	30.6	18.7	17.8	2.5		5.2	2.9		74 W
73 Ta		860.3*	821.0	728.6	200.1	181.0*	160.7*	132.8	128.4	41.7	33.0*	30.0*	18.0	17.2	2.5		5.3	2.6*		73 Ta
71 Lu		801.1*	763.1*	682.0*	184.5*	167.0*	149.4*	121.7*	117.9*	37.7*	30.9*	27.2*	16.2*	15.5*	1.3*		4.9*	2.5*		71 Lu
70 Yb		773.4*	735.7*	659.6*	177.5*	161.1*	144.7*	116.8*	113.2*	36.6*	29.9*	26.4*	15.2*	14.6*	1.0*			2.5*		70 Yb
68 Er		718.1*	682.8*	616.0*	163.8*	147.3*	132.2*	107.3*	104.2*	33.2*	26.9*	23.4*	13.4*	12.8*	0.3*		4.1*	2.4*		68 Er
67 Ho	4115.9	692.9*	657.4*	595.0*	157.3*	142.4*	129.0*	103.0*	100.0*	32.1*	26.6*	23.5*	12.7*	12.2*	0.1*			2.4*		67 Ho
66 Dy	3972.5	666.9*	632.3*	574.0*	150.9*	136.1*	123.7*	98.3*	95.6*	30.8*	25.3*	22.4*	11.9*	11.4*	(−0.2)*		2.9*	2.5*		66 Dy
65 Tb	[redacted]	642.4*	608.2*	553.8*	145.0*	130.9*	119.4*	94.1*	91.6*	29.3*	24.1*	21.8*	11.5*	11.1*			3.5*	2.4*		65 Tb
64 Gd	3711.9	617.4*	584.3*	533.7*	138.6*	124.6*	114.2*	89.8*	87.4*	28.3*	22.2*	20.2*	11.0*	10.6*			3.5	2.4*		64 Gd
63 Eu	3583.4	593.6*	561.3	514.1	132.9	119.5*	109.6*	85.8	83.5	26.9	21.9*	19.7*	10.4	10.1			2.9			63 Eu
62 Sm	3457.0	570.7*	539.0	495.0	127.1	114.3*	105.3*	81.9	79.9	25.8	20.2*	18.8*	10.1	9.8			3.0	2.3*		62 Sm
60 Nd	3214.2	525.8*	495.9	457.9	116.6	104.4*	96.4*	74.3	72.7	23.8	18.6*	17.2*	9.6	9.3			3.2	2.5*		60 Nd
59 Pr	3093.3	504.2*	474.7	439.4	111.4	99.0*	92.1*	70.1	68.6	22.5	18.0*	16.7*	8.8	8.5			2.6	2.2*		59 Pr
58 Ce	2972.2	483.0*	454.4	421.9	105.9	94.2*	87.8	66.7	65.4	21.7	17.0*	15.8*	8.7	8.5			3.1	2.2*		58 Ce
56 Ba	2756.4	442.3	414.7	386.7	95.4	84.9	79.4	58.8	57.7	18.8	15.3	14.4	7.1	6.9			2.1	2.4		56 Ba
55 Cs	2649.1	421.8	395.0	369.5	89.6	79.3	74.4	54.8	53.8	17.3	13.6	12.6	6.1	5.9			2.1	1.7		55 Cs
53 I	2448.3	382.6	357.6	336.0		69.0	64.8	46.8	46.0				4.4	4.2						53 I
52 Te	2345.0	364.1	339.6	320.1	74.5	64.4	60.5	43.2	42.5	12.7	8.8		3.4	3.3				0.4		52 Te
51 Sb	2241.7	346.1	321.9	304.3	69.3	59.9	56.4	39.5	38.8	11.1	7.2		2.3	2.2				0.1		51 Sb
47 Ag	1878.9	279.5	260.0	247.3	53.3	43.5	41.2	27.9	27.5	7.5	3.3		0.6							47 Ag

	K	L_{I}	L_{II}	L_{III}	M_{I}	M_{II}	M_{III}	M_{IV}	M_{V}	N_{I}	N_{II} N_{III}	N_{IV} N_{V}	N_{VI}	N_{VII}	O_{I}	$O_{\mathrm{II,\,III}}$	$O_{\mathrm{IV,\,V}}$	
45 Rh	1709.1	253.4	231.4	220.9	45.8	39.7	38.5	22.5	22.2	5.3	5.0	(−0.3)						45 Rh
42 Mo	1473.1	211.9	193.9	186.2	37.6	30.5	29.7	17.4	17.2	4.7	3.0	0.5						42 Mo
41 Nb	1401.3		181.4	174.4	35.1			15.0	14.9	4.0								41 Nb
40 Zr	1325.8		169.6	163.5	31.4			13.0		3.5								40 Zr
29 Cu	661.1		71.3	69.8		5.2												29 Cu
28 Ni	612.0		62.6	61.3		3.3												28 Ni
27 Co	568.9		59.7	58.5		5.3												27 Co
26 Fe	523.8		53.2	52.2		4.0												26 Fe
25 Mn	482.4		48.7	47.9		4.2												25 Mn
24 Cr	441.1		43.0	42.3		3.5												24 Cr
23 Va	402.3		38.2	37.6		2.6												23 Va
22 Ti	365.4		32.6	32.2		2.2												22 Ti
21 Sc	331.2		30.3	30.0		2.7												21 Sc
20 Ca	297.5		25.9	25.6		2.0												20 Ca
19 K	265.3		21.4	21.2		0.9												19 K
17 Cl	207.8		14.8	14.7		0.4												17 Cl
16 S	181.8		11.8			0.3												16 S
15 P	158.3		9.9			0.8												15 P
13 Al	114.7		5.2			0												13 Al
12 Mg	95.8		3.5															12 Mg

Experimental Arrangement for the Direct Determination of the Heat of Vaporization of Water.—The method described in this article is novel in not starting from the assumption that heat of vaporization is equal to the heat evolved when a given mass of vapor condenses into a liquid. Since it was primarily designed for a laboratory experiment, this is an important feature. Its simplicity gives it a strong appeal as an experiment for demonstration, and at the same time renders it capable of yielding results of fair accuracy. One disadvantage is that the heat of vaporization is not measured at normal boiling point; on the other hand there is compensating advantage in the fact that the method makes possible measurements at a variety of temperatures below boiling point. Two similar beakers of 250cc are partially filled with about the same quantity of water. Both beakers are heated until the temperature of the water rises to 70° or 80°C. Loss of heat takes place by conduction, by radiation, and by the evaporation from the surface of the water. If at the same given temperature the surface of one beaker is covered with a few drops of thin oil the loss of heat by evaporation is eliminated, and this beaker cools more slowly than the one not so treated. Essentially the method consists of counterpoising the two beakers on a balance of approximately .05 grams sensitivity, noting when the temperatures are the same, covering the surface of one beaker with oil, counterpoising once more, and allowing the water in the other beaker to evaporate until its temperature has fallen a definite amount, say 10° below the temperature of the other. To stop further evaporation the water in the second beaker is then covered with oil and, by weighing, the mass of water is found which has evaporated during the cooling process. Assuming that all of the excess heat lost in the one beaker as compared with the other was due to evaporation, the heat of vaporization is easily computed. The author points out a number of sources of error, chief among which is the fact that radiation losses from the two beakers are different because their mean temperatures during the period of evaporation are different. A method is suggested for eliminating this source of error. This is the well known procedure (ascribed to Rumford) of starting at a temperature as many degrees higher than the mean temperature of the protected beaker as the final temperature is below this point. Although this does not result in complete elimination of this source of error it does give acceptable results. A source of error is that the heat of vaporization at the lower temperature is higher, and that the specific heat of water is also about 2% greater, than at the normal boiling point. The author gives a detailed discussion of sources of error and their elimination and, in a table of experimental results, shows the possibilities of the method. [Method for the direct determination of the latent heat of vaporization of water. Paul Werner, ZS. f. den Physikalischen und Chemischen Unterricht, *36*, p. 30; 1923.]

PAUL E. KLOPSTEG

A NEW FORM OF PORTABLE ILLUMINOMETER

By Davis H. Tuck[1]

ABSTRACT

Description of a new form of portable illuminometer and accessories known by the trade name of the Holophane Lightmeter, Model *D*. The instrument has been in development since 1918 and has been manufactured by the Keuffel and Esser Company since 1922. Various models are in use by several laboratories for the measurement of (a) Illumination intensity; (b) Brightness; (c) Luminous intensity; (d) Luminous flux; (e) Reflection factor; (f) Absorption factor; (g) Transmission factor; (h) Special uses some of which are described.

There have been available for several years, semi-portable illuminometers such as the Sharp-Millar and Macbeth instruments, and the purpose of the new design was to incorporate the desirable features of these illuminometers and eliminate their bulk and weight.

At the outset, it was decided to use the inverse square or a modification of the inverse square method of varying the illumination on the cube from the standard lamp. The Lummer-Brodhun cube principle was chosen for the optical arrangement. The chief variations from the usual constructions are the mechanical arrangements of the battery, rheostat, switch and electrical measuring device into an integral part of the instrument.

The arrangement and construction of the instrument are shown in Figs. 1 and 2. Fig. 1 shows a cross section through the instrument in a plane at right angles to the axis of the telescope eye piece. Fig. 2 shows the complete assembly of the instrument.

The voltmeter indicates the potential across the terminals of the standard lamp. This potential is controlled by a circular rheostat which is operated by turning a knurled disc through a window in the rear end. The three cell dry battery is housed in the sliding tube which carries the standard lamp and is of the ordinary three element flash lamp type. All electrical connections between the lamp, switch, rheostat and voltmeter are made automatically. The standard lamp is centered in a hemisphere painted inside with a sphere paint and covered with a disc of blue glass ground on one side. The ground side is turned toward the lamp so that the exposed surface of the glass is smooth and can be

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easily kept clean. The bright spot observed through the eye piece is the result of the illumination produced by the ground blue glass on a piece of white diffusing glass held in the stationary cube mounting, and

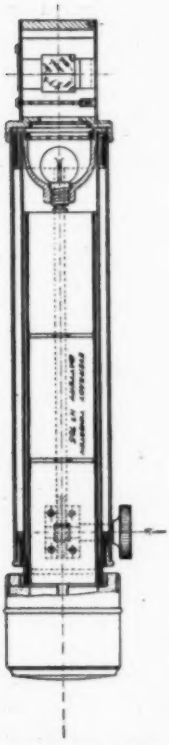

FIG. 1. *Cross section of Lightmeter showing construction.*

FIG. 2. *Photograph of Lightmeter completely assembled with battery, rheostat, switch and voltmeter.*

the distance between the ground surface of the blue glass and the nearest surface of the white diffusing glass is maintained constant for all instruments, as this distance determines the position of a reference

point on the scale of the instrument. The standard lamp is moved with reference to the white glass diffusing disc by a pinion which engages a rack on the battery housing on which housing is engraved the scale.

The standard lamp is a small tungsten filament lamp selected for flux output and seasoned. It is operated under its normal voltage so that its useful life[2] will be increased and the resulting yellow color is compensated for by the blue glass disc over the lamp housing.

Provision is made for the introduction of absorbing and color filters on either side of the cube through slots in the cube housing. The range of the instrument as ordinarily supplied has a ratio of maximum to minimum reading of 150,000, although the range can be further increased by the use of additional absorbing filters. Wratton filters cemented between glass and mounted in metal are used because of their comparative freedom from selective absorption. Wratton color filters are also used to give special color matches which may be required.

Outstanding mechanical features of the instrument are:

(1) Light weight (3 pounds including the battery)
(2) Dust tight cube housing
(3) No tools required for taking apart and assembling

Practical experience in making illumination measurements in the field has shown that a piece of white blotting paper with the matte surface used to reflect light is the most practical test plate. Ground milk glass test plates when carefully prepared, follow the law of diffuse reflection at larger angles to the direction of the incident light than does the blotting paper test plates, but these glass plates require frequent washing under unfavorable conditions and are easily broken. The blotting paper test plate is discarded when slightly soiled and its cheapness and reproductibility make it ideal for field measurements. In viewing the blotting paper test plate for making photometric measurements, the instrument should be held so that a line drawn through the axis of the telescope will be between 10 and 25 degrees from a line normal to the test plate.

ILLUMINATION MEASUREMENTS

The unit of illumination is the foot candle and is the illumination of a surface one foot distant from a one candlepower source. A surface having incident one lumen per square foot uniformly distributed has an illumination of one foot candle.

[2] The useful life of the lamp is its operation on the flat portion of its candlepower life curve.

In order to make measurements of foot candles incident on any plane of measurement, the test plate is placed on the plane of measurement and the test plate is viewed through the telescope. The voltmeter when set on the arrow indicates that the proper current is passing through the standard lamp. The lamp and battery housing is then moved in or out by means of a rack or pinion, until the two ellipses in the photometric field as viewed through the eye piece match in brightness. The scale reads directly in foot candles.

BRIGHTNESS MEASUREMENTS

The unit of brightness is the lambert which is defined as the brightness of a perfectly diffusing surface emitting one lumen per square centimeter. The one thousandth part of a lambert or the millilambert is the unit ordinarily used.

A perfectly diffusing surface emitting one lumen per square foot will have a brightness of 1.076 millilamberts. The Lightmeter will therefore read directly in millilamberts by multiplying the scale reading by 1.076 times the reflection coefficient of the test plate. In making brightness measurements, the test plate is not used, the surface whose brightness is required being viewed directly.

CANDLEPOWER MEASUREMENTS

Candlepower is the measure of luminous intensity. The present unit of luminous intensity is the international candle. The international candle is an arbitrary standard maintained at the National Bureau of Standards at Washington. To measure the candlepower of any source in a given direction, the illumination produced on the test plate at a measured distance from the light source in the given direction is determined and the candlepower computed by the relation $E = I/d^2$. Here E is the illumination in foot candles incident on the test plate, I is the luminous intensity in candles, and d is the normal distance from the source to the test plate.

COEFFICIENT OF DIFFUSE REFLECTION

The coefficient of diffuse reflection of a surface is the ratio of the flux diffusely reflected from the surface to the flux incident on the surface.

In order to determine the coefficient of diffuse reflection of a surface, the incident flux is measured by placing the test plate on the surface to be measured and measuring the foot candles (lumens per square

foot) incident on the surface. The test plate is then removed and a similar foot candle measurement made. The test plates supplied have a coefficient of reflection of 77 per cent when viewed at angles between 10 and 25 degrees. The coefficient of diffuse reflection of a surface is therefore—

$$\text{Coefficient of Reflection} = \frac{E_{\text{surface}}}{E_{\text{test plate}}} 0.77$$

FIG. 3. *Photograph of an observer measuring the illumination intensity in foot candles on a desk.*

In addition to the measurements described above, there have been several interesting applications of the Lightmeter, one of which is the measurement of brightness for machine printing of motion picture films. It has been the practice in motion picture laboratories to adjust the distance of the light source from the printing aperture by trial which involved lost time and lost film. By measuring the brightness of a glass

diffusing disk at the printing aperture, the correct position of the printing lamp can be quickly and accurately determined.

When taking motion picture negatives where the set involves considerable danger to the actors, a retake can often be made unnecessary by taking an illumination measurement before the picture is started to make certain that there is sufficient illumination. In making colored motion pictures, it is essential that the illumination be adequate and measurement before taking often saves the necessity of a retake.

A special application of the Lightmeter has been made by a manufacturer of radium luminous material. By measuring the brightness of samples of this material, its quality can be maintained to a standard. The brightness of this material varies from 1 to 100 microlamberts. A special form of instrument measuring 150 centimeters in length has been made so that the low brightness can be measured directly without the use of filters.

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Variation of Intensity of Helium Lines with Bombarding-Voltage, above the Ionizing Potential.—Two independent sets of experiments in each of which helium atoms were bombarded by electrons of energies adjusted between 27 volts and 90 volts, the emission spectrum being photographed. There are two entirely distinct helium spectra which are supposed to arise from two fundamentally different states of the atom; the relative intensity of these two spectra under various conditions is therefore of interest. The results of Udden and Jacobsen, which apparently are simply estimates made by visual inspection of plates, indicate that the orthohelium lines are much brighter than the parhelium lines at the lowest voltages used, but the latter increase steadily in relative intensity as the voltage is raised. The results of Bazzoni and Lay, which are measurements made by applying the neutral tinted wedge method of Nicholson and Merton, indicate on the contrary that there is no systematic difference between the two sets of lines at any voltage; but the orthohelium lines are found to rise in intensity from 25 to 35 volts, then *diminish* from 35 to 55 volts and rise again beyond 55; while the parhelium lines rise steadily in intensity over the whole range, but the rise is slower in the middle part. The helium used by Bazzoni and Lay was especially pure, and they record that they could not maintain a luminous discharge in it below the resonance potential, unless and until they admitted mercury vapor to the tube.—[C. B. Bazzoni and J. T. Lay, University of Pennsylvania, Phys. Rev. *23*, pp. 327-336; 1924. A. Udden and J. C. Jacobsen, ibid. pp. 322-326.] K. K. DARROW

BIBLIOGRAPHY FOR MEASURING INSTRUMENTS WITH SPECIAL REFERENCE TO INSTRUMENTS AND METHODS OF METROLOGY USED IN MECHANICAL ENGINEERING

COMPILED BY F. J. SCHLINK[1]

INTRODUCTORY NOTE

This bibliography, which was prepared during a number of years of specialization in metrology, particularly in the use of physical research methods in engineering measurements, seeks to cover only those fields which are not adequately taken care of in other reference sources. Furthermore, purely descriptive papers have generally been omitted, unless of unusual interest and value, since such papers are usually not of permanent importance.

Electrical instruments have been covered only to the extent of general papers or material directly applicable to mechanical engineering measurements. Some subjects have not been covered in detail because of the existence of extensive special bibliographies or sources of bibliographic information in those fields. Flow meters, for instance, are covered by a bibliography prepared under the direction of the A.S.M.E. Special Research Committee on Fluid Meters (Appendix D of Part 1, Fluid Meters, their theory and application.). A Committee on Instruments and Apparatus of the A.S.M.E. Power Test Codes Standing Committee will prepare a bibliography on Instruments especially applying to the field of power plant testing. For Aeronautic Instruments a bibliography is available from the National Advisory Committee for Aeronautics. In the case of limit gages and gaging practice, the Bureau of Standards can supply bibliographic information. The available bibliography on weighing devices has been much abridged on account of the specialized nature of this topic. The present bibliographer or the Bureau of Standards will be glad to supply additional references on this subject on request.

In the special study of measuring instruments the following journals should be particularly noted as sources of reference material:

Journal of the Optical Society and Review of Scientific Instruments
Journal of Scientific Instruments (British)
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[1] Member, Special Research Committee on Permanency and Accuracy of Indication of Engineering Instruments, American Society of Mechanical Engineers.

Elektrotechnische Zeitschrift (German)
Zeitschrift für Instrumentenkunde (German)
Journal de Physique (French)
Annual Reports and Collected Researches of the National Physical Laboratory (British)
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The assistance of Messrs. E. C. Laughlin and H. M. Roeser of the Bureau of Standards in preparing portions of this bibliography is gratefully acknowledged.

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AMERICAN ENGINEERING STANDARDS COMMITTEE,
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Liberation of Silver from Silver Bromide at the Rate of one Atom per Absorbed Quantum.—The experiments are performed on photographic plates consisting of emulsions of silver bromide, and require great care and labor in both of the essential measurements—that of the amount of light absorbed and that of the amount of silver liberated. The latter measurement forms a chemical problem which is solved by means which the reviewer is not qualified to judge; apparently the most delicate part of the work consisted in removing all the bromide and gelatine while losing none, or at worst a known amount of the silver. The former measurement is performed in two independent ways, by photometer measurements and by thermocouple measurements of the incident, reflected and transmitted light; the absolute intensity of the incident light is measured with a thermocouple calibrated upon a Hefner lamp, a certain value for the mechanical equivalent of light being assumed. Monochromatic light of wave lengths 365, 405, and 436 mμ was used. An essential question is, how much of the absorption occurs in the gelatine? To determine or at least assign a value for this, measurements of absorption were made on sheets of pure gelatine, as well as on the plates; it was assumed that the absorption of the plate at λ615 mμ, which is greater than that of the gelatine, is nevertheless entirely due to the gelatine, the difference occurring because the light is reflected to and fro in the gelatine from one particle of bromide to another, lengthening its path; the correction for shorter

waves is made by taking the observed values of absorption in pure gelatine and multiplying by the same factor. The difference between uncorrected and corrected values is taken to be the amount of light actually absorbed by silver bromide molecules, and is expressed in quanta and divided into the number of freed silver atoms. The quotient remains pretty close to unity so long as the number of quanta absorbed per cm^2 of film remains between 0.3 and $3 \cdot 10^{16}$, although the scattering of the individual data is considerable (1.08 to 0.82). As the amount of absorbed energy is increased the ratio drops off rapidly.—[J. Eggert and W. Noddack (Berlin) ZS. f. Phys., *20*, pp. 299-314; 1923.]

K. K. DARROW

Relativity—A Systematic Treatment of Einstein's Theory. By J. Rice, M.A., Senior Lecturer in Physics in the University of Liverpool, England. 389 pages. Longmans, Green & Co., 55 Fifth Ave., New York, $6.00.

This book gives a comprehensive and thorough treatment of relativity theory designed for the serious student of Physics and Mathematics who "is anxious to ascertain in a precise manner what changes this new idea is producing in the principles and content of physical science." (We quote from the preface). The physical bearings are therefore well emphasized.

The book is divided into three main parts besides a short introduction: Restricted Relativity, General Relativity and World Geometry. In the first part, the author has spared no pains to make the distinction between the Lorentz theory of variable mass and length and the relativity view. This part includes a clear discussion of relativity kinematics, the invariance of the electromagnetic equations, and a rather complete study of the use of tensor methods in the special theory. The treatment of proper time is unusually good.

In the discussion of the general theory in the second part, care is taken to clear up difficult points and guard the reader against common errors. The author has also indicated "precisely the points of similarity between the Tensor Analysis for transformations with variable coefficients employed in Part II."

Part III includes the cosmological speculations of Einstein and DeSitter, the theories of Weyl and Eddington, and finally Einstein's most recent generalization. A bibliography is appended.

Throughout the book, time is measured in $1/c$ seconds. The reader is frequently reminded of the convention whenever misunderstanding might arise in the interpretation of results (e.g. $m = E$ instead of the more usual $m = E/c^2$).

The clear and well-rounded presentation will make the book valuable to students of mathematical physics who care to give the subject the requisite thought.

D. E. RICHMOND

SOME ELECTRICAL AND OPTICAL PROPERTIES OF NEON GLOW LAMPS

ENOCH KARRER AND A. PORITSKY

The neon glow lamps have aroused considerable curiosity in the last few years. Some of their interesting characteristics have been described and many uses for them have been suggested. To these we shall scarcely make further mention, but we append a bibliography which may be of service in this connection. Our present purpose is to give some data bearing especially upon the relative intervals of light and darkness, and upon the distribution of luminosity over the potential cycle when the glow lamp is excited with alternating potential. We shall present some preliminary data too, bearing upon the efficiency of the lamp as a function of input wattage, and upon its spectrum. Upon these several aspects no observations seem to have been reported.

Most of the data that we give here pertain to two neon glow lamps, one (No. 1) of which was imported (an Osram Glimmlampe) and the other (No. 2) was an experimental lamp made locally.[1]

The electrodes in both lamps were of the same size and of a shape suggestive of an acorn (see Fig. 1)—one electrode (2) being the basal, the other (1) the apical end of the acorn.

We found that such a lamp may give very erratic behavior in its various characteristics when treated in various ways.[2]

When the apical electrode of lamp No. 2 was the cathode (i.e., the luminous electrode) in a direct current circuit the minimum potential required to produce glow (glow potential) is 155 volts, while the lowest voltage at which the glow will continue on reduction of potential (extinction potential) is 137*v*. When the basal electrode is cathode the glow potential is 165 and the extinction potential 150*v*. No auxiliary resistance is embodied in the lamp itself. A resistance of several

[1] This and several other lamps of a similar kind were made under the supervision of Mr. R. F. Strickland whose co-operation we wish to acknowledge. Our data cannot be taken as an adequate basis of comparing these lamps. Much greater liberty was taken with the experimental than with the commercial lamps.

[2] So much so that a large number of observations which we had made on a batch of glow lamps whose common characteristics we desired to establish, displayed large and unaccounted for discrepancies. We had planned to continue this study with an improved photometer especially designed for low brightness and with a sufficiently sensitive wattmeter for quite low wattage. Since our work has suffered interruptions we are giving the preliminary data on two lamps only which were observed somewhat more in detail.

hundred ohms was usually in series with the lamp but potential drop was always measured across the lamp only. In the first experiments the resistance had some inductance, which was later reduced to a negligible amount. In certain experiments when the lamp was connected directly to the terminals of a 50 *W* transformer no auxiliary resistance was inserted. The voltage across the lamp was in the latter case always varied by adjusting the voltage on the primary terminals of the transformer.

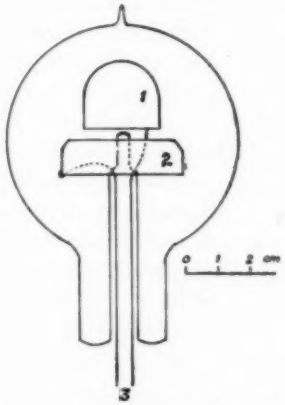


FIG. 1. *Sketch showing position and shape of electrodes in neonglow lamps. 1. apical electrode; 2 basal electrode; 3, leads.*

The voltage current characteristic of lamp No. 2 is given in Fig. 2 where curve 2 (crosses) is obtained when proceeding from the lowest to higher voltage, while curve 3 (dots) is obtained on decreasing the voltage. (Voltage along abscissae is relative.) How the light intensity varies with the current through the lamp is depicted in curve 1, Fig. 2. If we proceed further with these data we find that they show a rather large decrease of efficiency (relative intensity of light per watt) as the wattage is increased. During these observations the lamp had been allowed to arc several times. Also the readings were made in quick succession, and this we found later affected results in a manner as to augment the lag that is so clearly indicated in Fig. 2. For these reasons observations were repeated more slowly with this lamp using a noninductive resistance too. How the voltage-current has changed may be seen in Fig. 3, curve 1. The lag is appreciably less.

The slope of the curve is noticeably different. This is also the case with the curve 2 for relative intensity of light vs. current. Curve 3,

Fig. 3 indicates that the intensity of the light increases linearly with the wattage; while curve 4 shows that the efficiency (relative intensity per watt) may be independent of the wattage. For the observations

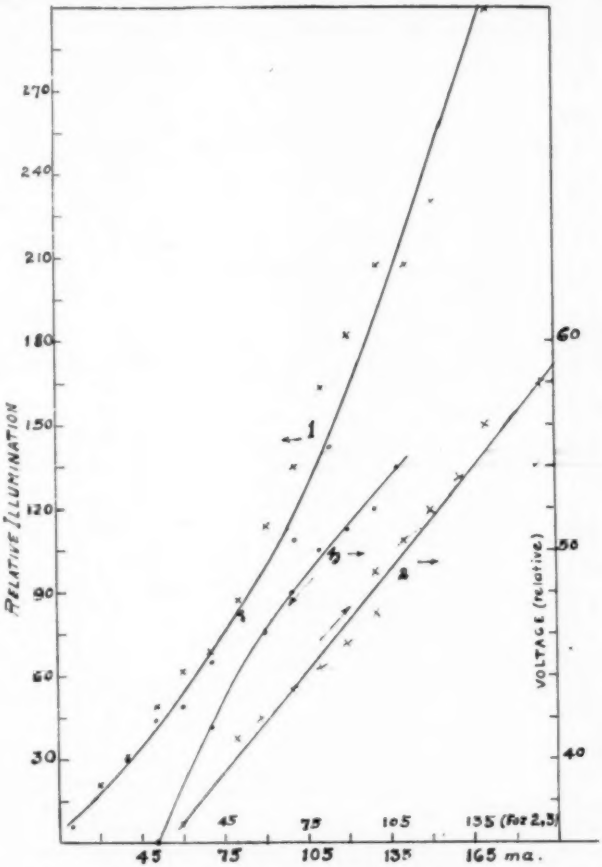


FIG. 2. *1 Relation between current through and illumination (relative) from lamp No. 2. Current—voltage characteristics, 2 (crosses) for increasing potential; 3 (dots) for decreasing potential; for volts multiply by about 3.8.*

taken on increasing wattage this is strictly so, although those (only a few points) made on decreasing wattage seem to indicate higher efficiency for lower wattage, as indicated by the dotted portion. We would be justified, however, in concluding that efficiency is independent

of wattage. To get better assurance on these points we repeated observations with lamp No.1 under identical conditions with the results plotted in Fig. 4. In this case the hysteresis is small. The current-voltage curve 1, is a straight line for the upper portion but is

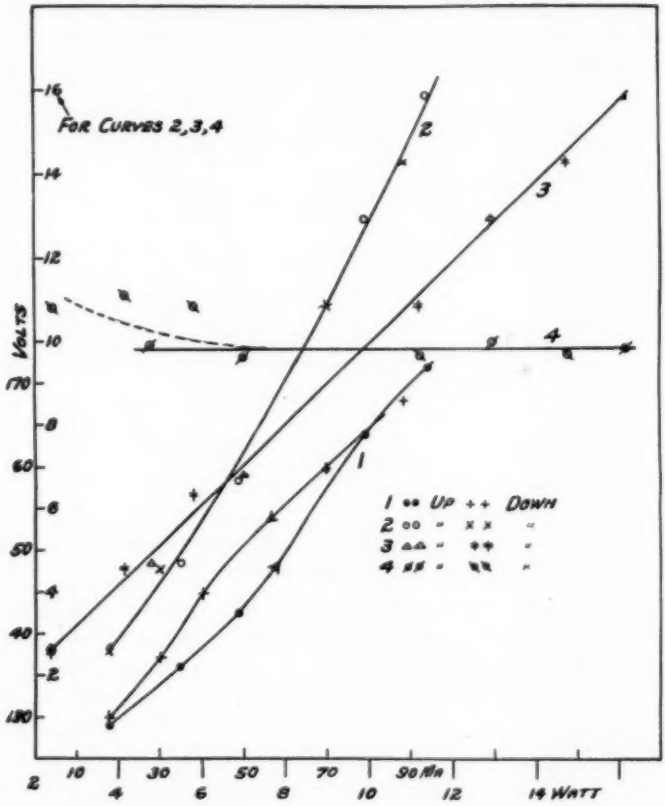


FIG. 3. *Characteristics of lamp No. 2 (repeated after lamp had arced).*
1. voltage vs. current (abscissae millamp. ordinates volts).
2. intensity (relative) vs. amperage (for mean spherical candles multiply by .065.)
3. intensity (rel) vs. wattage (for mean spherical candles multiply by .065).
4. efficiency (relative) vs. wattage (for candles per watt multiply by .065.)

slightly convex to the voltage axis for the lower portion. The intensity vs. current curve (2) is again almost a straight line; (concave to voltage axis) as is also the intensity vs. wattage. The efficiency (intensity ÷ wattage) is, however, not like that of the previous lamp, but shows a

slight increase with wattage, the rate of increase being more pronounced for the lowest wattage. However, there are several details to which attention should be directed before one attempts to compare too minutely such curves as these. It will be noted that curve 1, Fig. 4 is a straight line down to the voltage where one electrode becomes non-

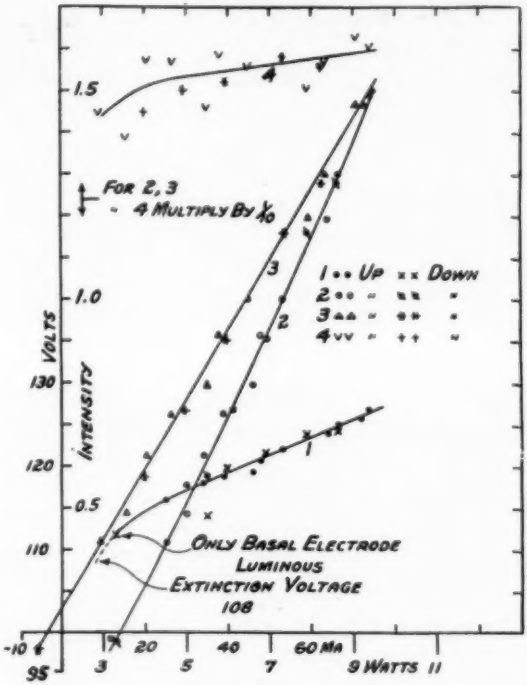


FIG. 4. *Characteristics of lamp No. 1.*

1. voltage vs. amperage.
2. intensity vs. amperage.
3. intensity vs. wattage
4. efficiency vs. wattage (multiply by 0.1 for candles/watt)

luminous. Some times, too, in lamps of this kind the spots of the electrodes which are first to become luminous are so situated that they cannot radiate effectively. One need not expect the efficiency of discharge lamps to vary with wattage regularly as is commonly the case with incandescent filament lamps. The distribution of radiant power among the various lines of the various series of the spectra of gases is not very well known; nor is it known just how the distribution may

change with current density for example. It has been found in case of the sodium vapor discharge lamp that the efficiency decreases with current density.[3]

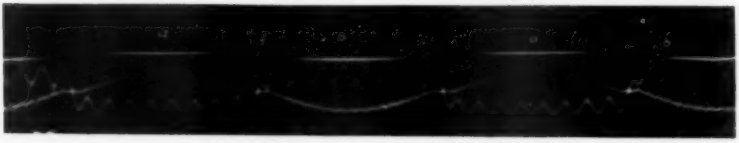


FIG. 5. *Oscillographic record of voltage (v) amperage (c), and illumination (a,b).*
213.6 volts, 140 milliamp. aa illumination from basal, bb from apical electrode; d, dark interval.

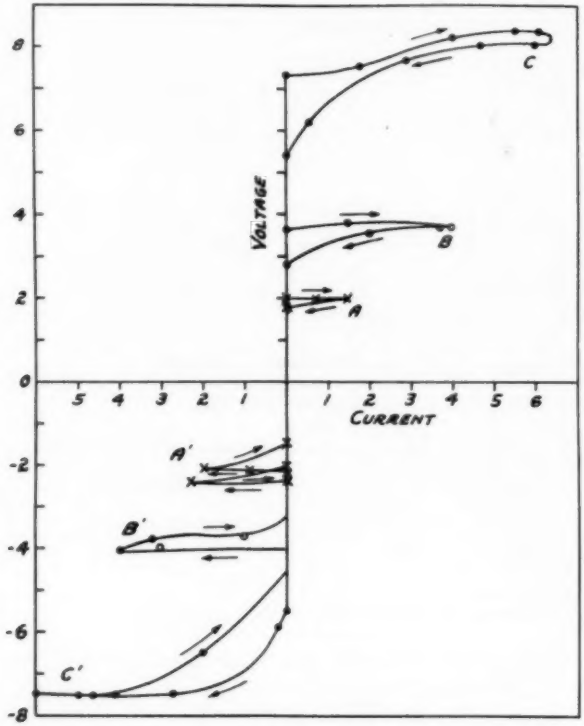


FIG. 6. *Relation between instantaneous current and potential. Data taken from oscillograms.*

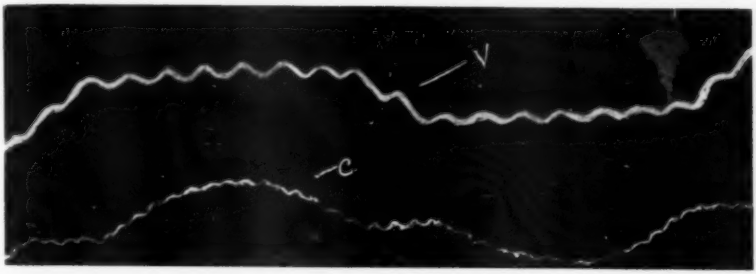
AA'	*131.2 v*	*10 ma.*
BB'	*139*	*23*
CC'	*257*	*138*

ordinates of AA' to be multiplied by 1.6 for comparison with others.

[3] Compton and Van Voorhis, Phys. Rev., p. 210; 1923.

The latter may be an unusual case since the radiation and ionization potentials are so low that the associated wave length of the radiation are in the photic region. Another point which must be taken into account when the wattage is increased by increasing the voltage, is the fact that the portion of the potential cycle during which light is produced becomes greater. Some observations upon this last point we shall give

A



B

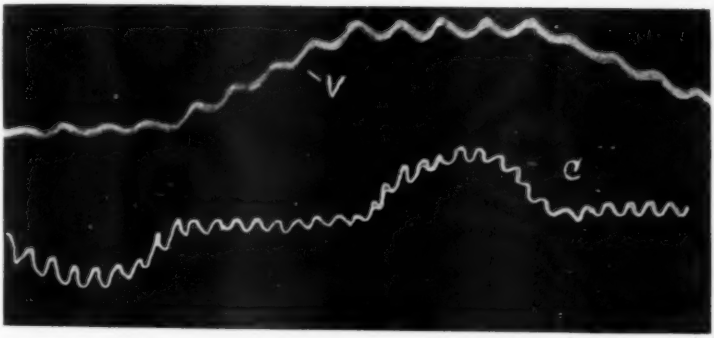


FIG. 7. *Oscillograms to show the relative increase of dark interval (i.e., no current) when voltage is decreased.*

A	*183 v.*	*52 ma*
B	*139 v.*	*20 ma*

(These curves being too faint for reproduction were retraced with some irregularities resulting. The relatively horizontal portion is the dark interval.)

after we have considered the oscillograms giving a record of current, voltage and in some cases illumination. A typical oscillogram for lamp No. 2 is reproduced in Fig. 5. The voltage (v) assumes a rather constant value after the current (c) has begun to flow. In general the current curve is steeper when the current is increasing than when decreasing. This is more evident in the curves of Fig. 6 which are

obtained from oscillograms for three values of voltage across the lamp (AA', 131.2; BB' 139; CC' 257). The greater abruptness of rise than of fall is brought out at the bends of the curves. The falling curve lies in general under the 'rising' curve for these instantaneous values just as in the voltage-current curve of Fig. 2. This is not always true, however, as some of our oscillograms indicate. It has been found by others[4] that there may be either positive or negative hysteresis in the voltage-current curves.

In the oscillogram of Fig. 5 the streaks marked "a" and "b" are a record of the instantaneous values of the illumination on the oscillo-

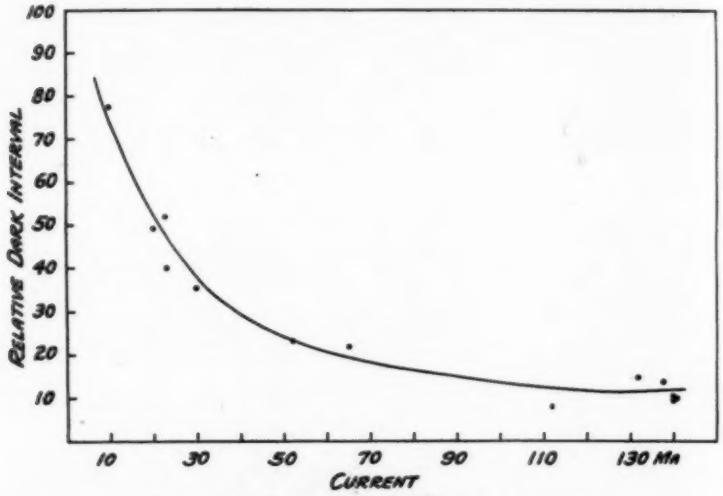


FIG. 8. *Relation between the dark interval and current.*

graph drum. This record was obtained by focusing an image of the luminous electrodes onto the drum, around which is secured the photographic film.[5] The broken line a is the record due to one electrode, and b that due to the other.

The interval (marked d) between any two consecutive dashes represents the interval of darkness, and corresponds to the interval when no current flows. When the voltage is decreased this interval of no current and of darkness is relatively increased as is shown in the oscillograms of Fig. 7 for A, 183 volts (v) and B, 139 volts. The interval of darkness is

[4] Ruettenauer, Zs. f. Physik, p. 33; 1923.

[5] The oscillograph used was an old type of the Siemens-Halske Company.

represented as a function of the current in Fig. 8. For the highest current allowable through the lamp this dark interval is about 10%, but becomes 80% at the point where the electrodes are just fully covered with glow. For commercial lamps of this kind one might expect a dark period of nearly 40%.

The instantaneous values of the photic power radiated during a cycle may be obtained by a study of the photographic density of the streaks *a* and *b* of the film from which the print of Fig. 5 was made. We were, however, able to measure the distribution of brightness through the cycle

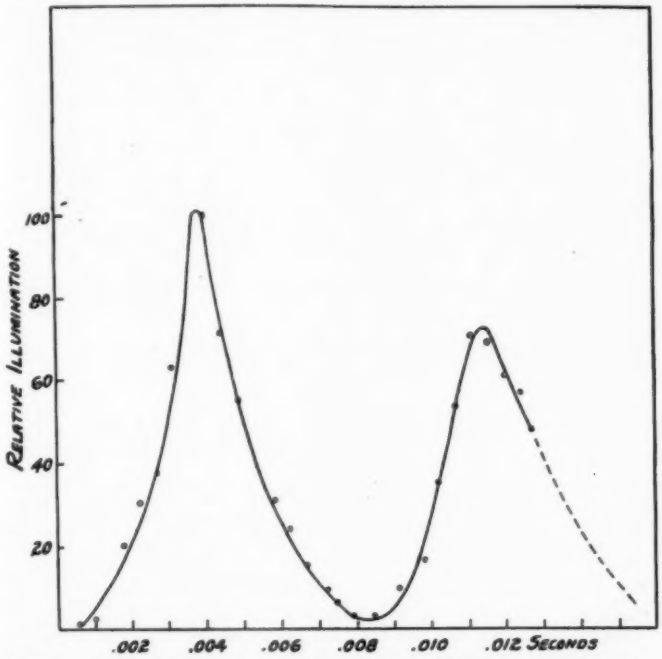


FIG. 9. *Distribution of the intensity (relative) of light through the potential cycle.*

directly by a special photometric set up to be described elsewhere.* The observations that are plotted in Fig. 9 were obtained under voltage and current conditions depicted in the oscillogram of Fig. 5. The resulting curves are a form of exponential curve, somewhat asymmetrical with respect to the maximum. As the current-voltage curves were seen to be steeper on ascent than on descent, so also are the brightness

*J. Fr. Inst. 198, p. 93, 1924.

curves. Certain aspects of relationship are more clearly brought out in Figs. 10 and 11. In Fig. 10 are plotted the instantaneous values of current voltage and photic power through a cycle.

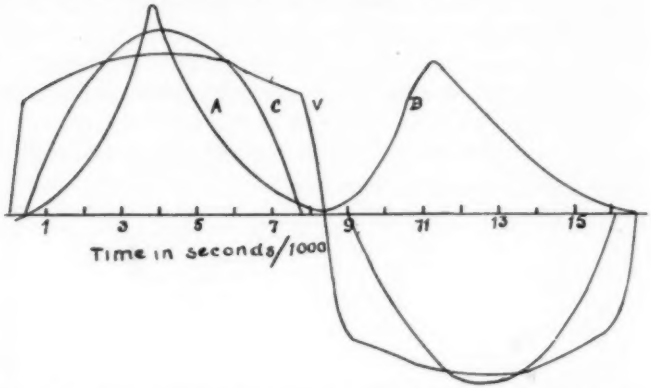


FIG. 10. *Curves showing the relative instantaneous values of voltage v, amperage c and illumination A.B. through the potential cycle.*

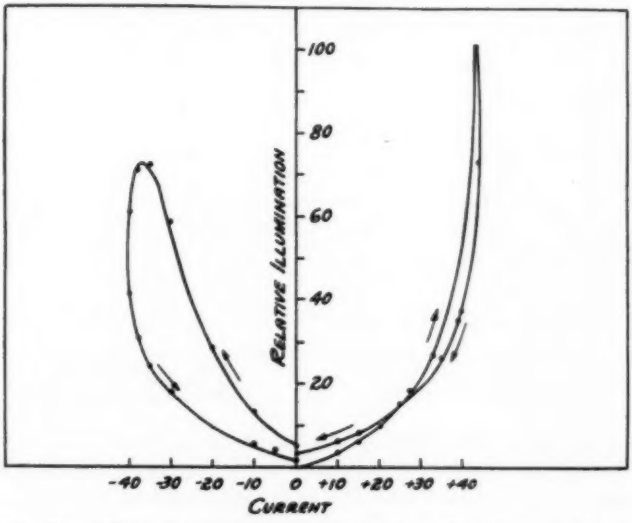


FIG. 11. *The instantaneous illumination plotted as a function of amperage.*

On Fig. 11 the photic power (relative) is plotted against the current for one cycle. The lag between current and illumination seems excessive especially in the second half of the cycle, where our photometric

measurements were less numerous. There does, however, appear to be some hysteresis like effect between the flow of electricity and the emission of light. If the lag were symmetrical and opposite in the two halves of the cycle, errors due to instrumental shift could be effectively invoked. Since our photometric measurements were not so numerous as desirable, we give this curve to show how the illumination and current are in a general way related through the potential cycle, without dwelling upon the apparent lag between them.

Among the various uses that have been suggested for neon glow lamps is that as an illuminant for the photographic dark room. From this viewpoint the spectrogram of Fig. 12 is of interest. The neon spectrum has been thoroughly analysed[6] and is known to consist of many series of lines. The first radiating potential of neon is about 11.8 volts[7] having associated with it radiation of wave length .105μ (far in the ultraphotic[8] region). The first ionization potential is about 16.7 volts giving the spectral term of .074μ. It must be clear, therefore, that there must be present ultraviolet radiation whenever the luminous glow appears. There are also reasons for believing that this ultraphotic is more intense than the photic radiation (principally the orange in this case). The short wave radiation is strongly absorbed by the bulb glass, thus limiting the spectrogram in that region. The lime glass used in lamp bulbs begins to absorb rather intensely at .34μ and increasingly so until at .300μ the transmission is only a few per cent. Although any one line of neon is very faint and scarcely comparable with any one of the significant mercury lines, yet, the large number of lines of neon somewhat makes up for this in certain parts of the spectrum. Our conclusion is, therefore, that the spectrum of neon can not be encouraging to those who have suggested the neon glow lamp as a dark room illuminant,—but rather is suggestive of other applications for it where use may be made of a weak ultraviolet spectrum.

NATIONAL LAMP WORKS OF G. E. CO.,
CLEVELAND, OHIO.

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[6] Paschen Ann. d Physik, *63*, p. 201; 1920; Sommerfield, Ann. d. Physik, *63*, p. 221; 1920. Paschen and Gatze, Seriengesetze der Linienspektren, J. Springer, 1922.

[7] Hughes, Bul. N. R. Council 2, p. 83; 1921.

[8] See Science *60*, p. 30. 1924.

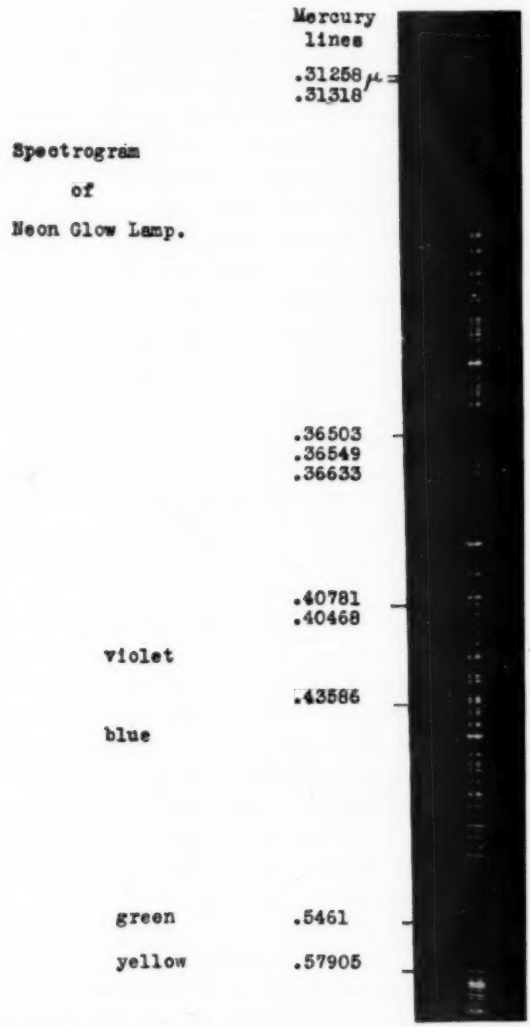

FIG. 12. *Spectrogram, showing the limits and relative intensity of the neon lines through lime glass.*

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Cause of the "Low-voltage" Arc in Helium Gas.—Investigating the arc which is maintained in helium gas between a heated cathode and a cold anode, with a voltage far below the resonance potential of the gas, (as low as 10.5), Bär, Laue, and Meyer found that it could not be maintained when the tube was connected directly to the terminals of a 13-volt battery without any resistance coils, galvanometers, etc. in series. This suggested to them that the arc is maintained by virtue of high frequency oscillations which carry the voltage above the resonance potential at short intervals. This they tested in numerous ways; they connected an inductance between the tube and the battery, and could maintain the arc; they short circuited the tube with a $2\mu f$ capacity and the arc ceased; they heard a humming sound in a telephone connected in series with the arc, and transformed it into a musical tone by connecting a small ($0.5\ \mu f$) capacity across the tube, under which circumstances the arc operated at 6 volts; they used a rotating mirror on this 6-volt arc and found it broken into successive discharges separated by dark intervals. By using a three-electrode tube as a peak voltmeter in one of the accepted ways, they showed that the peak voltage always exceeded 20 during the maintenance of the arc, even when the steady applied voltage was as low as 12 volts. Various experiments were then performed with circuits in which capacities, inductances, and resistances were connected in a two-mesh network to the tube; the values of the adjunct capacities and inductances influenced the frequency, but its

value could not be calculated by assuming these to constitute the entire reactance of the oscillating circuit, and depended on filament current, arc current, electrode separation and helium pressure. Oscillations could be obtained with no adjunct capacity whatever and inductances of the order of 0.1 henry; they were above audiofrequency, and were measured in some cases as $3.10^4 - 3.10^5$ cycles. These oscillations were independently observed by Eckart and Compton, who by employing a high voltage (120-volt) battery (with a series resistance sufficient to reduce the DC voltage across the tube below the resonance potential) could obtain them even when the inductance "was as nearly as possible eliminated." In addition they observed oscillations when the DC voltage across the tube exceeded the ionizing potential, but there were no oscillations when it lay between the ionizing and resonance potentials. The German observers also detected no oscillations when the DC voltage exceeded the resonance potential. Eckart and Compton observed the effect in mercury vapor, but not in hydrogen or nitrogen. They conceive that when ionization commences, the space charge near the cathode becomes positive and permits so large a current to flow that the potential drop across the series resistance rises so high as to cut down the voltage across the tube below the resonance potential; ionization then ceases, the positive space charge is replaced by a negative one, the current and the voltage across the series resistance sink, and the cycle starts anew.—[R. Bär, M. v. Laue, E. Meyer (Zürich and Berlin); ZS. f. Phys. *20*, pp. 83-95; 1923. C. Eckart, K. T. Compton (Princeton); Science *59*, pp. 166-168; 1924.]

K. K DARROW

Do X-rays suffer a Change in Wave Length on Reflection from Crystal Planes?—Suppose that they do, and that Bragg's relation $\sin\theta = \lambda/2d$ nevertheless connects the angle of reflection θ with the incident wave length λ. Suppose further that a beam of x-rays is dispersed by a revolving crystal and received on a plate in the usual way; if a substance having a critical absorption wave length λ_0 is placed in the beam between crystal and source the absorption edge will appear in a certain place on the plate; if the substance is placed in the beam between crystal and plate the absorption edge will be displaced by an amount corresponding to the wave length shift of the reflected rays. This can easily be seen by considering the component of the primary beam (before incidence on the crystal) having λ_1 $(\lambda_0 > \lambda_1 > \lambda_0 + \Delta\lambda$, where $\Delta\lambda$ represents the wave length shift); it will be within the absorption band if the absorbent is placed between crystal and source, but not if it is placed between crystal and plate. This is Kulenkampff's argument. He looked for the effect, using crystals of calcite and of rocksalt and absorbing plates of Fe, Ni and Zn and working in the vicinity of the *K* absorption edge; there was no indication whatever of it.—[H. Kulenkampff, Munich; ZS. f. Physik, pp. 17-19; 1923]

K. K. DARROW

AN INCANDESCENT LAMP WITH A QUARTZ WINDOW

By Donald C. Stockbarger

As is well known, there is a demand for a source of ultraviolet whose spectrum is continuous throughout the range of the ordinary quartz spectrograph. Perhaps the most convenient source is the quartz incandescent lamp, but the cost of such a lamp is comparatively high, and an experienced quartz blower's services are required whenever repairs have to be made. That repairs are unavoidable is evident when we consider that the filament must be operated at an abnormally high temperature in order to extend the spectrum far into the ultraviolet. Not only is the life of the filament short, but the darkening of the bulb becomes serious.

G. Gehlhoff[1] has described a glass incandescent lamp having a quartz window cemented to the end of a tube. Such a construction has obvious advantages, but generally it is found better to avoid the use of cements as far as possible, especially in apparatus which should be heated strongly during fabrication.

J. Franck and W. Grotrian[2] have described a modification of the Gehlhoff lamp whose lower part, together with the filament, can be removed readily. The ground joint is sealed by means of sealing wax which is kept cool by immersing the lower part of the lamp in water.

It is the purpose of this paper to describe a lamp which, it is hoped, will be of interest to anyone requiring a source of continuous ultraviolet. It is similar to the Gehlhoff lamp except in the construction of the side tube.

The side tube, which is 5/8 of an inch inside diameter by about 6 inches long, is a graded lime glass-to-quartz joint on the end of which is sealed a ground and polished plane fused quartz window. This tube is sealed on the side of a special 6.6 ampere series type, nitrogen filled, tungsten lamp having twice the usual length of filament. The filament leads have been bent in such a way as to shorten the distance between filament and window, and to eliminate any possibility of reflection by the leads into the tube, as is seen in the accompanying photographs.

In preliminary experiments a similar lamp, without the side tube, operated on 7.3 amperes for more than 80 hours without burning out.

[1] G. Gehlhoff, ZS. f. techn. Physik *1*, p. 224; 1920.

[2] J. Franck and W. Grotrian, ZS. f. techn. Physik *3*, p. 195; 1922.

It is expected that a life of approximately 100 hours can be realized, which would be sufficient for several months of service under ordinary conditions.

The following tests on the lamp were carried out with a Hilger quartz spectrograph using an orthonon plate. The distance of the filament from the slit was 13 1/2 inches. The slit width was 0.01 mm. The exposures were 1, 2 and 6 minutes at currents of 7.35, 7.35 and 7.30 amperes, respectively. The ultraviolet limits were found to be 2350, 2300 and 2250 A., respectively. In each case, the intensity over the last 50 A was weak, but the photographic blackening over the remainder of the spectrum was sufficient.

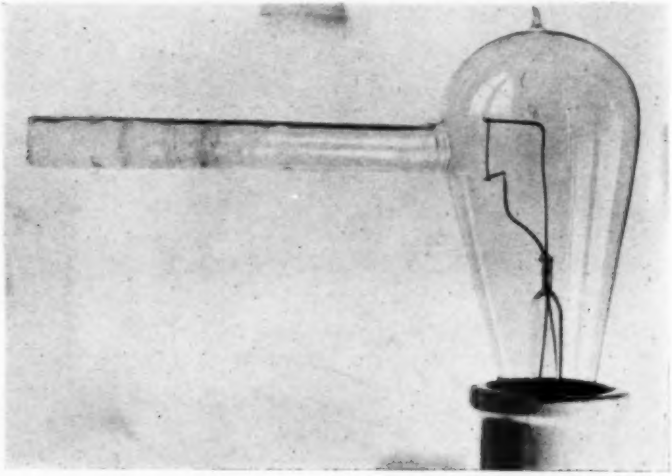

FIG. 1. *Side view, showing side tube with fused-on window.*

The lamp can be operated on a 25 volt source or on the ordinary 115 volt power supply if the voltage is constant.

The advantages of the lamp lie in its permanence, portability, and ease of repair. The tube can be sealed on any standard lamp having the proper size and shape of filament, so that special construction is unnecessary. Side tubes of the graded joint type, of almost any size, can be made for sealing on pyrex, lime glass or lead glass. The length of the tube is sufficient to prevent much, if any, blackening of the window.

The side tube was made by the Cooper Hewitt Electric Company, Hoboken, N. J.; and the special lamps used in the preliminary studies,

as well as the final assembled lamp, were made by the Hygrade Lamp Company, Salem, Mass., to both of whom I am greatly indebted for their generosity. The work was done through the courtesy and coopera-

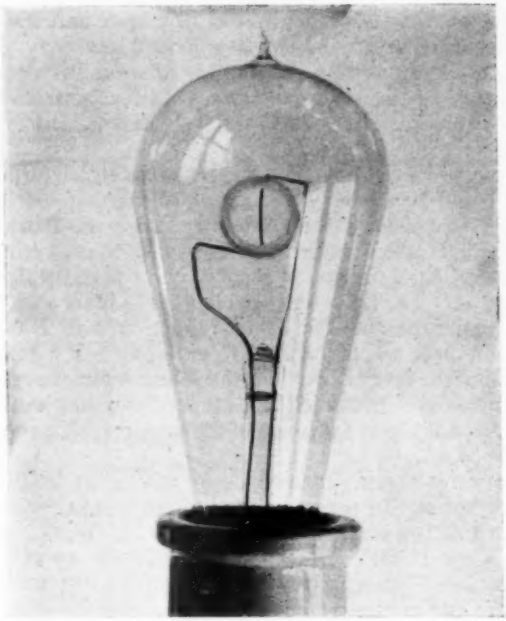

FIG. 2. *Front view, showing shape of leads and position of filament.*

tion of Mr. L. J. Buttolph and Mr. J. R. Fuller of the above mentioned companies.

ROGERS LABORATORY OF PHYSICS,
ELECTROCHEMICAL DIVISION,
MASS. INSTITUTE OF TECHNOLOGY.

Notes on Silvering of Glass by the Formaldehyde Process.—This process is economical and very rapid. Only one bath is required. It has been employed for mirrors more than 20 cm in diameter.

The process discussed is that of M. J. Izarn, who simplified the procedure suggested by M. Louis Mumière. One begins by making sure that the distilled water to be used in making the bath and for washing the glass does not give a precipitate with nitrate of silver and that it is neutral.

The ammoniacal solution of silver nitrate is prepared by placing 15 grams of silver nitrate in a glass stoppered liter flask partly filled with distilled water. Commercial concentrated ammonia (about 22° Baume) is added. About 17 cc is required. A little less is used and then the remainder added drop at a time until the precipitate is just dissolved. The solution is then watered to 1 liter and its temperature noted and recorded on the label. If preferred, one may allow a slight precipitate to remain and on pouring out the quantity desired for use at any time may add a very small crystal of silver nitrate, agitating the solution until the precipitate disappears. In the making of the solution colorless crystals should be used.

To find the proper proportion of the commercial formaldehyde to use a trial should be made. Place about 13 drops in a small porcelain dish and add 15 cc of the silver solution agitating the dish and watching the reduction. If the amount of formaldehyde is correct the liquid takes on a rose violet hue in a few seconds and has the typical aspect of a colloidal solution. The color quickly changes to blue, then to iron gray, and finally to bright metallic. The reduction, which takes about a minute, is complete when the liquid contains clearly defined clumps. There is insufficient formaldehyde if the solution remains gray or yellow and appears muddy. Too much of it causes the reduction to take place too rapidly, and the silver does not take on a metallic appearance and is easily rubbed off.

The operation is easy with new glass or with glass that has just been polished with rouge. In resilvering it suffices to wash with nitric acid, rubbing with a cotton pad and washing well with distilled water leaving the glass immersed. Old surfaces may be difficult to silver. If rouge cannot be employed one may treat the glass with a mixture of potassium permanganate and concentrated sulphuric acid, aqua regia, and finally with a solution of potash.

The dish in which the silvering is to be done may be washed at the same time. The fingers should not touch the cleaned surfaces.

Temperature is important, the proper value for the glass seeming to be that marked on the label of the silver solution.

The thickness of the silver deposited is about 50 millimicrons.

If lines are to be ruled in the silver this should be done immediately after silvering.

A chamois skin on a pad of cotton is first used in polishing the surface. The effect of this is to compress the silver and give it a solidity that will withstand the final polishing. A second pad charged with English rouge it then used with great care.

For semi-silvering it is well to reduce the speed of the reaction by adding a volume of distilled water 1/10 the volume of the bath employed.—[M. A. Cotton, Revue d'Optique, 3, pp. 57-64; 1924.]

G. W. MOFFITT

A CURVATURE GAUGE

By Chester K. Wentworth

The gauge described below has been used by the writer to measure the radius of curvature of corners and edges of pebbles and cobbles in the course of his study of their shapes. An earlier form of the instrument was referred to briefly in 1922,[1] but since this reference is probably not generally known to instrument users and since the gauge has been constructed in a simplified form it is thought desirable to publish the following note.

The instrument consists essentially of a wedge which slides in a split block, Fig. 1. The bearing surface of the wedge is the upper margin of the long central slot which slides on two cylindrical brass bushings. These are carried between the two parts of the split block on the screws which are shown in the face of the instrument. The lower and parallel part of the wedge below the bushings serves only for stiffness and is not involved in the measuring function of the instrument. The upper margins of the two parts of the split block are formed to a cuspate curve as shown in Fig. 1. The edge of the pebble to be

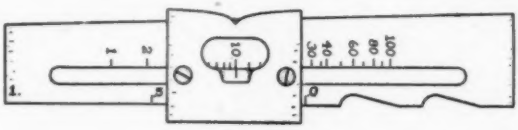

Fig. 1

measured is applied to this curve so that the axis of the nearly cylindrical surface is normal to the plane of the curve and with the pebble surface in contact with it on both sides of the cusp. The wedge is then moved toward its small end until its upper surface comes in contact with the surface of the pebble as a third point and the radius of curvature in millimeters is read on the scale as shown.

The unique feature of the instrument lies in the form of the cuspate profile. Fig. 2 illustrates the derivation of this curve. Circles a, b, c, d, and e, having their centers on the line MN, have radii in geometrical series so that

$$\frac{r_a}{r_b} = \frac{r_b}{r_c} = \frac{r_c}{r_d} = \frac{r_d}{r_e} \text{ etc} = K$$

[1] Wentworth, C. K., U. S. G. S. Bull. 730 C, pp. 94-95; 1922

Circles *a*, *b*, *c*, *d*, and *e* are so arranged on line *MN* that their nearest points to *M* lie on a uniform scale and their distances from *M* are thus in arithmetical series. The curves *HI* and *JK* are the envelopes of this series of circles. From the derivation it will be seen that they have the property that a series of tangent circles having their radii in geometrical series will have their nearest points to a point *M* arranged in arithmetical series along the line *MN*.

The use of such a profile for the saddle points of the gauge together with the uniform slope of the wedge achieves the result that constant ratio differences between the radii of cylindrical surfaces are read by constant linear amounts on the wedge scale. The wedge scale of radii is therefore a logarithmic one which may be read at all points with the same relative or percentage accuracy. As in all logarithmic scales the range of similar accuracy in reading is very greatly increased.

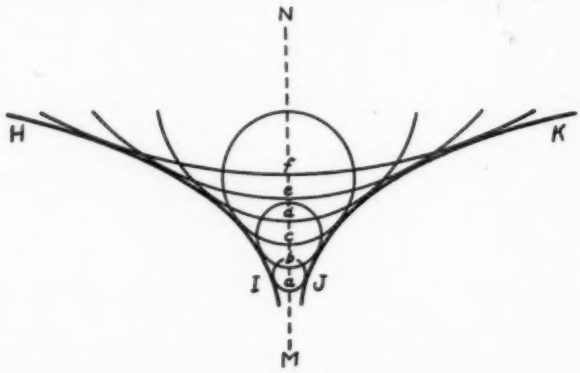

FIG. 2

An improvement, not embodied in the instrument shown in Fig. 1, would be to so skew the screws and bushings that the upper surface of the wedge would be at right angles to the line of symmetry of the cuspate profile. In the instrument shown the upper surface of the wedge has the attitude imposed by its slope and does not impinge on the precise lower point of the pebble which is being measured. There is thus introduced a slight error in the meaurement of larger radii especially if the curves be other than circular. Though the profile was constructed with considerable care it was found to have slight deviations from the theoretical form and the actual scale marked on the instrument was determined by calibration.

Because of the large range of the scale and the consequent low slope of the wedge the instrument is not one of great accuracy. With clean cylindrical surfaces radii of curvature may be measured within perhaps 3% commonly. Estimation on the scale is somewhat more accurate than the friction contact and it is possible that greater resulting accuracy would be attained if the scale were made slightly shorter and the wedge correspondingly steeper for the same profile.

In summary the gauge has the following characteristics:—

1. Accuracy of 3 to 5% in measuring radii of curvature from 1 to 100 millimeters.
2. Uniform percentage accuracy over the entire range.
3. Simple, staunch construction. (Designed for field use in studies of stream and beach pebbles.)
4. Lack of glass or other fragile working parts.

BISHOP MUSEUM,
HONOLULU, T. H.

A New Method of Demonstrating Equipotential Surfaces in the Electromagnetic Field.—The authors sought to develop a method which would be convincing to students in its demonstration of equipotential surfaces. The method consists of producing an alternating magnetic field and exploring this field by means of a small search coil connected to a telephone headset either directly or through vacuum tube amplifiers. As a specific illustration the authors describe first the mapping of equipotential lines in a plane surface consisting of a sheet of paper mounted on a drawing board. Two laminated bar magnets are placed so that their axes lie in one of the bisecting lines of the paper with the poles slightly projecting over the edge of the paper. Magnetizing coils are provided so that like poles face each other. The exploring coil is conveniently mounted on a circular card arranged to facilitate plotting the line. When the axis of the coil coincides with the line of force the response in the telephone is maximum. When the coil is turned perpendicular to this position the tone becomes either zero or a very perceptible minimum. A 3-stage amplifier permits exploration of the field by the search coil to more than a meter from the poles of the exciting magnets. The authors describe the exact procedure in plotting the equipotential lines and discuss the possible criticisms of the method. To prove that the field consists of equipotential surfaces rather than lines they suggest cutting templets, one straight edge of which lies in the axis of symmetry of the field, while the curved edge is defined by one of the equipotential lines intersecting the axis. If this templet is so placed that the straight edge corresponding with the axis of symmetry is located in its proper position on the axis, the search

coil placed anywhere on the curved edge will indicate the minimum as the templet is revolved about the axis. The demonstration thus shows that equipotential surfaces are surfaces of revolution.—[A new method for representing equipotential surfaces in electromagnetic fields. Martin Beghardt und Erich Günther, ZS. f. den Physikalischen und Chemischen Unterricht, *36*, p. 1; 1923.]

PAUL E. KLOPSTEG

X-RAYS.—By G. W. C. Kaye, 4th Revised Edition. Pp. XXII+320, Longmans, Green & Co., 1923. $5.00

The purpose and scope of this book, as stated in the author's preface, is to present the main principles of x-ray radiology and enough of the physics involved to give students of physics, and particularly medical students, an understanding of the development of the subject since Roentgen's discovery of x-rays in 1895. It is not intended to serve as a treatise or handbook on x-rays. The author has maintained in the present fourth edition, which involves considerable revision to bring the book up to date, the same purpose and with the same commendable success as in previous editions.

About half of the volume is devoted to a discussion of the technique of practical radiology and x-ray equipment. A short but important appendix on x-ray protection with practical recommendations, which are of utmost value to the practical roentgenologist, has been added.

The book begins with a brief discussion of the phenomena of electrical discharge tubes which led to the discovery of x-rays. This is followed by chapters on the characteristics of x-ray tubes, methods of measuring x-rays, and the more important physical properties of x-rays as such. Practical applications are then considered. These are followed by a chapter on the diffraction of x-rays by crystals which describes briefly Laue's theory of x-ray diffraction in crystals and the important developments of x-ray spectroscopy which have resulted from the pioneer work of the Braggs and Moseley. The book closes with a short chapter on the nature of x-rays. Several tables of physical constants relating to x-ray data and practical suggestions relative to high vacuua and electrical insulation are included in the appendix.

A fuller discussion of the epoch-making work of Bohr, which is barely mentioned, would have been welcome and one of the gaps in the periodic table of the elements to which reference is made has been filled by the discovery of hafnium by the methods of x-ray spectroscopy since the new edition went to press. These are to be considered, however, only as evidence of the rapid growth of the subject in many directions, to which the author refers in the preface to this edition, and do not affect the purpose or usefulness of this excellent volume.

F. L. HUNT

AN ELECTROSTATIC VOLTAGE MULTIPLIER

By Alfred W. Simon

If a variable condenser is charged to a potential V_1 when its capacity is C_1 and if its capacity is then changed to C_2, the voltage across the condenser becomes V_2, where V_2 is given by:

$$V_2 = C_1V_1/C_2$$

This can be demonstrated very nicely by connecting a Braun electrostatic voltmeter, or an electroscope, across a variable air condenser. It also serves as a convenient method of charging an electroscope, or any system of small capacity, to a definite high voltage from a low voltage source. Used in this way, the variable condenser becomes essentially an electrostatic voltage multiplier.

The transformation ratio r is given by the equation:

$$r = (C_1 + C_3) / (C_2 + C_3)$$

where C_3 is the capacity of the system to be raised to high potential. Obviously, if r is to be large C_1+C_3 must be large compared to C_2+C_3; or, since C_2 will usually be small, this means that C_1 will have to be large compared to C_3. In general, therefore, the maximum capacity of the variable condenser will have to be much larger than that of the system to be charged, and this fact limits the application of this method to charging systems of small capacity, since otherwise the required variable condenser becomes too unwieldy.

However, a method of charging capacities even larger than that of the variable condenser immediately suggests itself. If the system to be charged is connected to the variable condenser only at the instant when the voltage across the latter has reached the proper (high) value and is disconnected while the variable condenser is charged and varied, charge can be continually transferred to C_3 until the voltage of C_3 becomes equal to the potential V_2 given by:

$$V_2 = C_1V_0 / C_2 \qquad (4)$$

where V_0 is the voltage to which C_1 is originally raised, i.e., the voltage of the source.

The action of such a device can readily be followed theoretically. If V_n represents the potential of C_3 after n transfers (or cycles) we have:

$$V_n = (C_3V_{n-1} + C_1V_0) / (C_2 + C_3) \qquad (5)$$

By substituting for V_{n-1} its value in terms of V_{n-2}, and so on, it can be readily verified that V_n is given by:

$$V_n = C_1V_0K\,(1 + K + K^2 + \cdots + K^{n-1}) / C_3 \qquad (6)$$

where K has been substituted for the expression:

$$K = C_2 / (C_2 + C_3)$$

Summing the series we obtain:

$$V_n = C_1V_0K\,(1 - K^n) / C_3\,(1 - K)\,, \qquad (7)$$

from which

$$V_\infty = C_1V_0K / C_3\,(1 - K) = C_1V_0 / C_2 \qquad (8)$$

This agrees with the result already deduced above, namely equation (4).

It is readily seen that C_1/C_2 plays the role of the transformation ratio rather than $(C_1+C_3)/(C_2+C_3)$ as was the case for the preceding type of voltage multiplier. In other words, the capacity of the system to be charged does not affect the transformation ratio.

From (4), (7), and (8), it is obvious that V_n can also be written

$$V_n = V_\infty\,(1 - K^n) \qquad (9)$$

$$= V_\infty\,(1 - e^{\beta n})\,, \qquad (10)$$

where

$$\beta = \log_e K\,.$$

If V_n is plotted for various (integral) values of n, the points will fall on the exponential curve given by equation (10).

Solving (9) for n we obtain:

$$n = \log\,(1 - V_n / V_\infty) / \log K$$

From the last equation the number of turns required to produce any given value of the ratio V_n/V_∞ can be calculated: in particular, the number of turns required to raise the potential of C_3 to 99% of its final value.

The ideal efficiency of a machine of this type in charging a condenser can be shown to be approximately 2/3. In the ideal case, the only energy lost is that lost when C_2 and C_3 share their charges. If Δl_n represents the energy lost at the nth sharing, we have:

$$\Delta l_n = C_2C_3\,(V_\infty - V_{n-1})^2 / 2(C_2 + C_3)\,.$$

The total energy, E_n lost after n sharings is:

$$E_n = \sum_1^n C_2C_3\,(V_\infty - V_{n-1})^2 / 2(C_2 + C_3)$$

Substituting for V_{n-1} its value, $V_\infty(1-K^{n-1})$, and summing the resulting series, we obtain:

$$E_n = C_2V_\infty^2\,K\,(1 - K^{2n}) / 2\,(1 - K^2)$$

from which:

$$E_\infty = C_3 V_\infty^2 (C_2 + C_3) / 2(C_2 + 2C_3)$$

When the machine is used to charge C_3, the output O is given by:

$$O = C_3 V_\infty^2 / 2$$

hence the ideal efficiency e is:

$$e = (C_2 + 2C_3) / (2C_2 + 3C_3) \qquad (14)$$

This reduces to 2/3 when C_2 is small compared to C_3.

Of course, after C_3 is once charged, the ideal efficiency becomes very nearly equal to unity, the machine now serving only to balance the loss due to leakage.

As a concrete example of the application of this theory, suppose a system of 500 cm capacity is charged by means of a variable condenser, for which C_1 is 1000 cm and C_2 is 50 cm, and a source of potential of 100 volts.

From equation (8) we find V_∞ to be 2000 volts.,

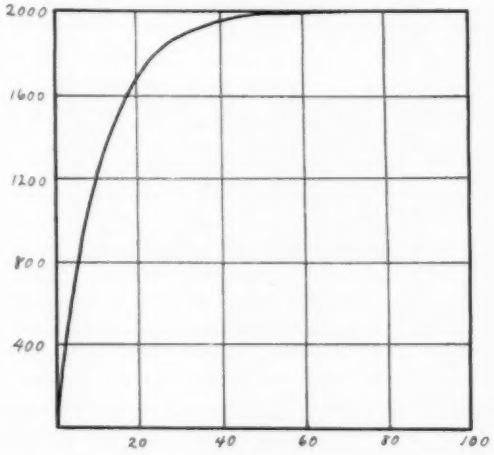

FIG. 1

From equation (9) the values of V_n corresponding to various values of n can be calculated. This has been done and the results plotted in Fig. 1.

It is seen that 100 turns suffice to make V_n practically equal to 2000 volts. If the machine makes 20 revolutions per second, 5 seconds are required to produce this voltage across 500 cm of capacity, and this would correspond to an average current of:

$$500 \times 2000 \times 10^6 / 9 \times 10^{11} \times 5 = 2/9 \text{ microamperes.}$$

From equation (14) the ideal efficiency is easily found to be 65.6%.

A device making use of these principles has already been described by Brenzinger[1] and also by Regener.[2]

In Brenzinger's machine the condenser takes the form of two parallel plates, the change in capacity being effected by varying the distance between the plates. Regener's machine is essentially the same except that one of the plates is a semi-conductor.

An improved machine, which was designed not only to charge up but to maintain the electrode of an ionization chamber at a constant potential of some 2000 volts, is one built in this laboratory. In this machine (Fig. 2) the variable capacity takes the form of a rotary plate, air condenser, of which the fixed plates are supported on bakelite pillars, while the rotating plates are carried by a steel shaft and balanced by

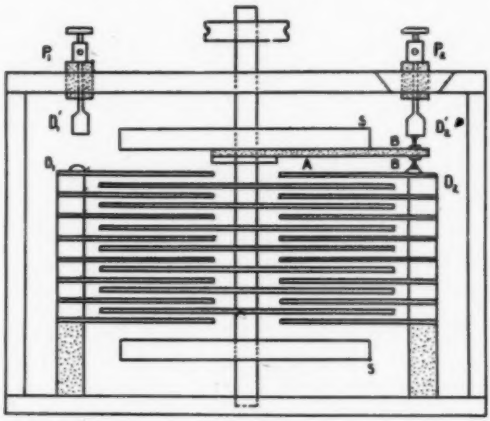

FIG. 2

means of two brass sectors S so that they can be rotated at high speed. The shaft also carries a bakelite arm A, at the outer end of which there is mounted an amber insulated brushholder carrying the contact brushes B, B. On the top of the fixed plates there are two studs D_1 D_2, 180° apart, opposite two corresponding insulated studs D'_1 D'_2, which end in binding posts P_1 P_2. These studs are so placed that the brushes B, B establish contact between P_1 and D_1 at the instant when the capacity of the condenser is a maximum, and between P_2 and D_2 when the capacity is at, or near, the minimum. In addition the system

[1] Strahlentherapie, *16*, 1923.
[2] Zt. f. techn. *3*, p. 220; 1922.

P_2 D_2' is mounted on a slider, which can be displaced over a short range in the direction of rotation by means of an adjustable screw, the corresponding stud D_2 being lengthened in proportion, so that it is possible to vary the capacity C_2 corresponding to the instant of contact. This serves as a fine adjustment for the voltage. The coarse adjustment is made by varying the line voltage. The shaft is motor-driven and kept in operation continuously when it is desired to maintain a given system at a constant potential.

Needless to say, the rotating plates are made the grounded set, since this makes it possible to construct the case, bearings, shaft, etc. entirely of metal, and leaves only the contact brushes, the two studs D_1' D_2', and the fixed plates to be insulated. This is readily accomplished by means of bakelite or amber in the way already described.

In conclusion I wish to thank Professor Arthur H. Compton for the interest he has shown in this work, and Mr. Ralph D. Bennett for his help in the construction of the machine.

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A Band in the Iodine Spectrum, ascribed to Electrons attaching Themselves to neutral Iodine Atoms.—Franck has suggested that when a free electron attaches itself to a neutral electronegative atom, there is radiated a quantum of energy equal to the "electron affinity" of the atom (in appropriate units) augmented by the kinetic energy of the electron just before it was drawn to the atom. Gerlach and Gromann have discovered a band in the ultraviolet spectrum of iodine vapor excited in a quartz tube by the electrodeless discharge (to avoid contamination of the vapor); this band has a sharp edge on the side towards longer waves, at 3460 A, and is without discernible structure; it becomes more conspicuous as the temperature is raised and the molecular band spectrum of the iodine vapor yields to the atomic line spectrum, its position remaining unaffected. Assuming that its long wave edge corresponds to the "electron affinity" of the atom (without supplementary kinetic energy) we get 81.8 ± 0.2 cal./gm mol. for this quantity, which agrees remarkably well with the values calculated from the constants of solid KI and solid NaI by Born's theory. The breadth of the band (not clearly defined) indicates a maximum supplementary energy not exceeding 0.2 volt. It seems probable that another band near 4800 which fades out along with the molecular spectrum, is due to the electron affinity of the I_2 molecule. Other bands indicate a vibrational frequency of 215 cm^{-1}. A new band spectrum emitted by mercury vapor in a quartz tube excited by the electrodeless discharge, is ascribed to Hg_2 molecules, the previously-known band spectrum of "mercury" being

assigned to HgNa molecules due to reactions with the walls of glass tubes.—[W. Gerlach, F. Gromann; ZS. f. Phys. *18*, pp. 239-248, 1923; Phys. ZS. *24*, pp. 467-469, 1923]

K. K. DARROW

Excitation of Spectra of Ionized A, Kr and Xe by Electron-Bombardment and by the High Frequency Electrodeless Discharge.—Exciting these gases in narrow tubes by the field of a high-frequency current through a coil around the tube, the Blochs and Dejardin observe four spectra appearing in succession as the voltage is increased; at first the arc spectrum, then the first spark spectrum appearing near the edge of the tube and gradually creeping to the center, and so forth. Bombarding the atoms of these gases by electrons from a hot filament, Dejardin observes that the arc spectrum appears at 15.2, 12.7, 10.9 volts, respectively (with an uncertainty of 0.2 volt); the first of the additional spectra at 34.0, 28.25, 24.25 (uncertainty of 0.5); the second of these at 70, 59, 51 (uncertainty of 2 volts). Still others lines of the third of the high excitation spectra appear to require still higher voltages. It appears therefore that the spectra of once, twice, and thrice ionized atoms of the three gases have been produced, and the critical potentials for single, double, and triple ionization determined. (The agreement between his and Hertz's recent value for the first ionizing-potential of argon is excellent). [L. Bloch, E. Bloch, G. Dejardin; C. R. *178*, pp. 766-769, 1069-1071; 1924.]

K. K. DARROW

Experiments indicating Expulsion of H-nuclei from Atom-Nuclei of Light Elements.—Kirsch and Pettersson enclose radium emanation in capillary tubes of glass or silica, part of each tube being covered internally with metal foil or an oxide film, while the rest of the tube is bare; the numbers of long-range particles emitted from the two parts of such a tube are counted (by the scintillation method) and the difference is considered to consist of long-range particles expelled from the metal atoms. The observations upon Al agree well with those of Rutherford and Chadwick; observations on Sc, V, Co and As gave negative results (i.e. proving that not more than 1/20 as many particles of range 17 cm can be expelled from any of these as from Al). The behavior of the tubes indicated, however, that H-nuclei of range 12 cm were expelled from silicon atoms. The silica tubes were therefore replaced by metal troughs covered with lids of very thin copper foil, and the H particles issuing through these were counted, additional absorbing layers being inserted in their paths so as to study the distribution-in-range after the manner of Rutherford. The results with salts of Be and Mg and with elementary Si indicated that H-nuclei are expelled from these atoms with ranges of 18, 13 and 12 cm respectively. [G. Kirsch and H. Pettersson, Vienna; Phil. Mag. *47*, pp. 500-512; 1924.]

K. K. DARROW

A VISUAL METHOD OF MEASURING SHORT TIME INTERVALS*

By Herbert Grove Dorsey

The operation of this machine is based on the principle of persistence of vision enabling one to read the position of a flash of light in a Geissler tube wherever it may occur while in rapid rotation. A disc carries a small Geissler tube back of a narrow radial slot. Slip rings and brushes make connection with the tube and the disc is rotated by a motor. Concentric with the disc is a graduated circular scale, and the flash of light being practically instantaneous gives a readable indication at almost any speed of rotation. More than one flash may be read during a single revolution and with slight practice the positions of three flashes may be easily read to half divisions on a scale 20 cm in diameter divided into 100 parts. This gives a method of measuring the time interval between the start and finish of one or two events occurring at unexpected times.

The Geissler tube may be operated by a spark coil operated by a hand key opening the primary circuit or if from a distance by a relay. It may be operated direct from an oscillatory circuit with thermionic tubes, and the latter may be operated by a trigger tube or the circuit may be kept oscillating during any interval which is to be measured. In this case a series of flashes extending over a definite portion of the scale will be shown. If the frequency of oscillation is high enough, say 10,000 cycles per second or more, the flashes appear as a continuous band of light, the beginning and end of which is easily read on the dial. Still another method has been tried in which an amplifier circuit is supplied with a known constant frequency, as from a tuning fork, and the circuit kept closed during the interval to be measured. In this case there will be twice as many sparks as cycles duration, and for a thousand cycle fork each spark would represent .0005 second. The speed of rotation need not be known since the accuracy will all depend upon that of the fork. From 3 to 10 sparks may be easily counted with certainty.

If the Geissler tube illumination is sufficiently actinic or if the tube is replaced by a spark gap between metals the sparks may be recorded photographically.

If a period of time longer than a single revolution is to be measured a second wheel may be geared to the first, and run, say one tenth as fast and the position of the flashes noted on both dials.

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The machine shown in the photograph was built for a special purpose in which the dial rotates 240 times per minute or four times per second and is graduated to 100 divisions on the inner dial, each division corresponding to .0025 second so that the instrument reads fathoms directly when used for determining ocean depths by the echo method.

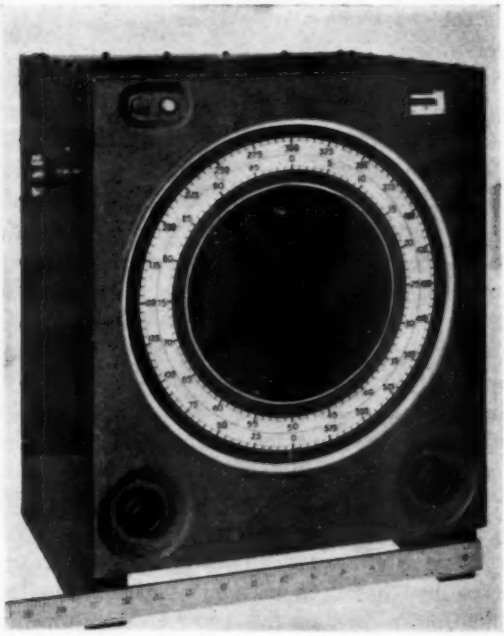

FIG. 1. *Fathometer.*

In this machine a frequency meter of the reed type is used to indicate the speed of the machine which may be regulated by a potentiometer tap for the armature voltage and an adjustable resistance in the field. The frequency meter is operated by alternating current of 30 cycles taken from slip rings on the armature of the D.C. motor running at 1800 rpm. In this machine the signal is sent into the water when the Geissler tube on the disc goes through zero of the scale so that the flash due to the echo gives the reading direct in fathoms.

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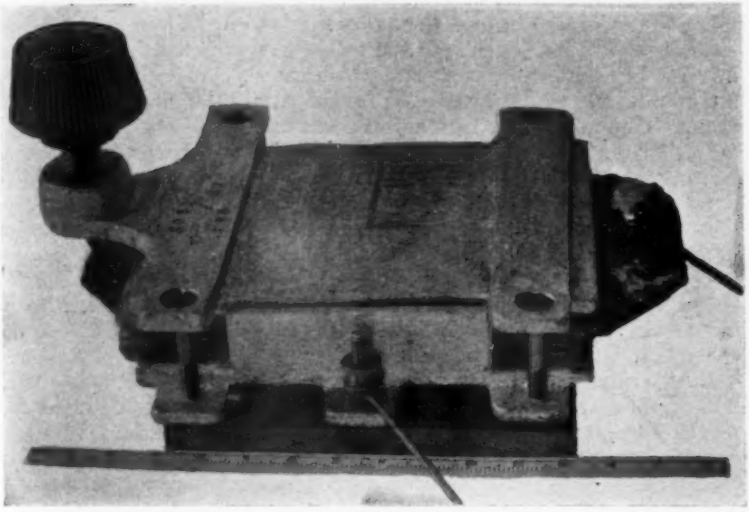
A VARIABLE MICA CONDENSER

By Herbert Grove Dorsey

A new type of Variable Mica Condenser which has proved quite useful in tuning electrical circuits is shown in the photograph. A number of plates of copper foil and phosphor bronze are alternated with sheets of mica between, the copper projecting at one end and the phosphor bronze plates at the other end. The plates are all assembled between two aluminum castings which are clamped together by screws. The projecting ends of the copper plates are all soldered together



forming one terminal and the phosphor bronze plates have their edges placed in a straight line at an angle of about 15 degrees so that a screw having a 30 degree conical point at its end will, when screwed forward, touch the plates successively, connecting two or more in parallel as may be desired. The aluminum plates and screw form the other terminal.

A convenient small condenser has about 18 pairs of plates 32×45 mm separated by mica $40 \times 50 \times 0.1$ mm with capacity of about 0.0002 μf minimum to about 0.018 μf maximum. A larger size has about 20 pairs of plates 70×50 mm separated by mica 70×100 by 0.1 mm and has a capacity from .001 to 0.1 μf.

These condensers are especially useful for resonating circuits where it is not necessary to know the exact capacity and where the tuning can be changed by steps. Since the capacity remains constant wherever set, it may easily be measured by a bridge when necessary. A small condenser may be used in parallel with a larger one so that values smaller than the individual steps can be obtained or the condenser may be made with steps of almost any amount by using different thicknesses of mica.

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Intensity of Rays Scattered from a Rocksalt Crystal in Various Directions.—In making measurements on x-ray scattering the first precaution is to avoid or correct for the reinforced scattering which takes place in certain directions owing to parallel atom-layers in the substance. "Amorphous" substances have been used, but it appears increasingly doubtful whether any substance, even a liquid, is truly amorphous and without vestiges of regularity in the arrangement of its atoms. One might also take an excellent crystal, locate the directions in which reinforced scattering occurs, and avoid these directions in making measurements; and by making such measurements it would be possible to make interpolations for the directions in which there is reinforced scattering, and so determine the value which the scattering in these directions would have but for the reinforcement. This is done in the experiments of Jauncey and May, because by virtue of a certain geometrical relation it is possible to determine the scattering coefficient directly from these interpolated values without knowing the absorption coefficient, which would be essential in interpreting values of scattered intensity in any other directions. In their experiments the x-ray beam falls on the front face of a crystal lamina about 1 mm thick, cut parallel to the 100 planes; the angle of incidence is given various values a, and the intensity of the ray scattered at angle $2a$ and emerging through the rear face of the crystal is determined by the interpolation method. The intensity of the primary beam is also measured, so that the values given for scattering coefficient are absolute. The scattering begins abruptly at about 5°, shoots up very rapidly to a maximum near 15°, descends to a minimum at 100° and rises again as $2a$ approaches 180°. The maximum is double, which the authors ascribe to the heterogeneity of the x-ray beam used (rays from a Mo target filtered through Zr). From about 60° onward the deviation from the well-known Thomson curve (based on the assumption of electrons vibrating in unison with the waves) is comparatively small, though unmistakable; the situation is about the same for the theories based on Compton's assumption of elastic collisions between quanta and electrons. [G. E. M. Jauncey and H. L. May, Washington University; Phys. Rev., *23*, pp. 128-136; 1924.]

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